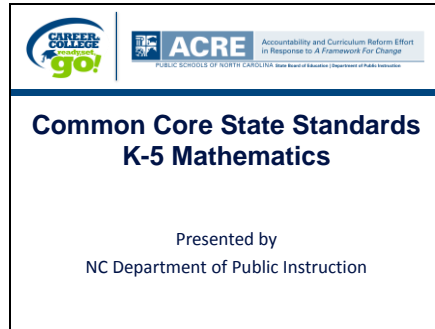
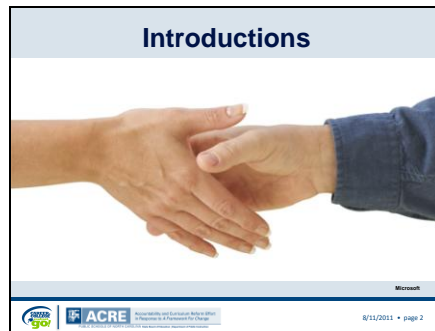


Slide 1



Welcome

Slide 2



Get to know your audience.  
How many are K-2 teachers, 3-5 teachers,  
coaches, administrators, etc...?  
Introduce yourself at your table.

Slide 3



- Parking Lot: Use the post-its and the parking lot to place questions you may have.
- Breaks: We will have a morning and afternoon break; restrooms are (give location)
- Technology: Please limit technology use to session activities; please check cell phones not to make sure they are on vibrate or silent
- Session Materials: Power point will be provided; because some of the nature of some activities, we will wait until the end of the presentation to give you this. Electronic copies of handouts will also be available.

Slide 4

### Norms

- Listen as an Ally
- Value Differences
- Maintain Professionalism
- Participate Actively



Accountability and Curriculum Policy Office  
In Response to Assessment for Change

8/11/2011 • page 4



Is there anything we need to add or change?  
Thumbs up if you can live with these norms.

Slide 5

### Timeline for Common Core Mathematics Implementation

Common Core State Standards Adopted June, 2010

Year	Standards To Be Taught	Standards To Be Assessed
2010 – 2011	2003 NCSCOS	2003 NCSCOS
2011 – 2012	2003 NCSCOS	2003 NCSCOS
2012 – 2013	CCSS	CCSS



Accountability and Curriculum Policy Office  
In Response to Assessment for Change

8/11/2011 • page 5

Timeline for implementation and assessment  
for Common Core State Standards



Slide 6

### Reading the Grade Level Standards

**Number and Operations in Base-Ten** 3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g.,  $9 \times 80$ ,  $5 \times 60$ ) using strategies based on place value and properties of operations.



Accountability and Curriculum Policy Office  
In Response to Assessment for Change

8/11/2011 • page 6

How many of you have reviewed the CCSS for mathematics?

• **Domains** are larger groups of related standards. Standards from different domains may sometimes be closely related. Unlike the 2003 NCSCOS, these will change throughout K-8

• **Bold Heading of Cluster** always look at bold heading when reading standard

• **Standards** define what students should understand and be able to do.

• **Clusters** summarize groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

• Coherent big ideas with a progression of learning vs. isolated skills that don't seem to connect to one another

Slide 7

Sample Crosswalk	
NC SCOs	Common Core
Standard	Standard
Test of objective	Test of objective
<p>Develop number sense for whole numbers through 9,999</p> <p>a) Conceptual model, number word, and number using a variety of representations</p> <p>b) Build understanding of place value (ones through thousands)</p> <p>c) Compare and order</p>	<p>Use place value understanding and properties of operations to perform multi-digit arithmetic</p> <p>Use place value understanding to round multi-digit numbers to the nearest 10 or 100</p>
<p>Develop fluency with multi-digit addition and subtraction through 9,999 using</p> <p>a) Strategies for adding and subtracting numbers</p> <p>b) Estimation of sums and differences to appropriate situations</p> <p>c) Relationships between operations</p>	<p>Use place value understanding and properties of operations to perform multi-digit arithmetic</p> <p>Fluently add and subtract within 100 using strategies and algorithms (including place value, properties of operations, and the relationship between addition and subtraction)</p>
<p>Develop fluency with multi-digit addition and subtraction through 9,999 using</p> <p>a) Strategies for adding and subtracting numbers</p> <p>b) Estimation of sums and differences to appropriate situations</p> <p>c) Relationships between operations</p>	<p>Use place value understanding and properties of operations to perform multi-digit arithmetic</p> <p>Fluently add and subtract within 100 using strategies and algorithms (including place value, properties of operations, and the relationship between addition and subtraction)</p>

Hyperlink included for access to crosswalk documents

Slide 8

# CAUTION!!

CONTENT APPEARING TO BE THE SAME MAY ACTUALLY BE DIFFERENT!!

The CCSS Requires **CLOSE** Reading!!!

Standards may seem similar but have very different expectations

Slide 9

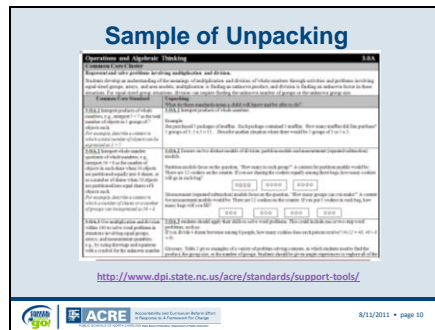
## Instructional Support Tools

### Unpacked Content

- A response, for each standard, to the question "What does this standard mean?"
- The unpacked content is text that describes carefully and specifically what the standards mean a child will now, understand and be able to do and explains the different knowledge or skills that constitute that standard.

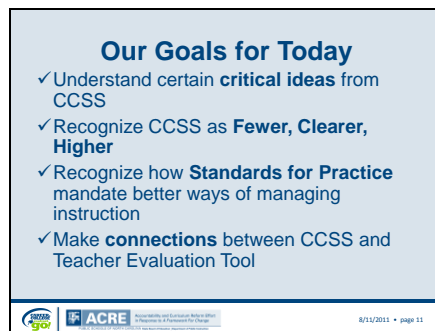
Documents continue to be revised as feedback from the field is given. The next revision is expected to be posted in August.

Slide 10



Hyperlink included for access to unpacking documents

Slide 11



- Three targeted critical areas are for this session 1. Algebra, 2. Composing and decomposing Number, 3. Fractions
- The Mathematical Practices are critical- they will make a difference in instruction
- As we progress through session attend to relationships to teacher evaluations -this will prompt teachers to move forward with new standards

Slide 12



- These new Standards are not just a rearrangement of content. Over the last 2 decades there has been an enormous amount of research about how students learn and how teachers 'Manage instruction'.
- CCSS are Fewer, Clearer and higher – focused standards
- Kathy Richardson- Asks: "What task can I give instead of how can I explain it?" Focus is on understanding rather than appearing to know- an "illusion of learning"
- John Van de Walle- Before, during and After (when the students share their thinking and strategies and the teacher questions) – Teaching by telling doesn't work

Slide 13

**ALGEBRA**

*Algebra is a  
generalization of  
arithmetic.*

Marilyn Burns

ACRE Accountability and Curriculum Policy Office  
In Response to a Request for Change  
8/12/2011 • page 13

We will first look at the foundations for algebra as happens in K-5. Read quote.

Slide 14

**"If there is a problem  
with algebra in your  
high schools, then you  
have to fix it in K-4."**

Kathy Richardson, NCTM, 2004

ACRE Accountability and Curriculum Policy Office  
In Response to a Request for Change  
8/12/2011 • page 14

Reflect on quote

Slide 15

**Common Core State Standards**

Standards are organized into  
Domains, K-5

*Operations and Algebraic  
Thinking*

ACRE Accountability and Curriculum Policy Office  
In Response to a Request for Change  
8/12/2011 • page 15

Operations and Algebraic Thinking is one of the Domains

Slide 16

**Operations and Algebraic Thinking** **K.OA**

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

- K.OA.3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g.,  $5 = 2 + 3$  and  $5 = 4 + 1$ ).

**NOTE:** Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.  
(page 11)

8/12/2011 • page 16

- This is the first standard we refer to in the power point. Review parts of the standards – **domain, bold heading of cluster and the standards**. Use vocabulary to get participants familiar with the new terms.
- The clearer higher standards in CCSS *explicitly* address equality in FIRST grade, however the concept of equality is first explored in Kindergarten as they decompose numbers. Equality is one of the mathematical properties that is fundamental in understanding mathematics.

Slide 17

**“On and Off”**

- Let's work on the number 7.
- Drop 7 counters over a piece of paper.
- How many counters land on the paper? off the paper?
- Record your work.
- Repeat 12 times.

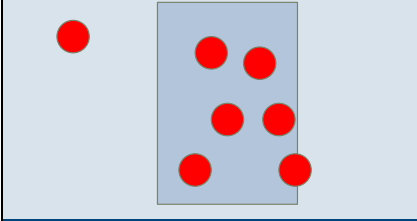
8/12/2011 • page 17

Any counters work, pennies, Cheerios, two color counters are a variation.

What were the strengths of those tasks?

Slide 18

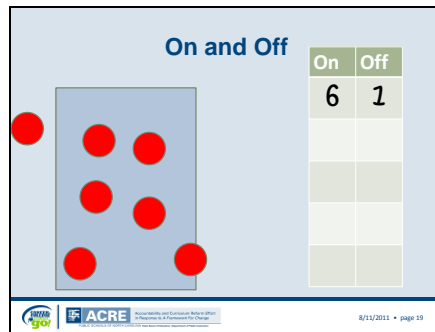
**On and Off**



8/12/2011 • page 18

Visual representation of task

Slide 19



Visual representation of task

Slide 20

**Turn and Talk**

- What did you notice?
- How many ways could you make 7?
- Did some seem alike? different?
- Did you always have 7 no matter how many were on or off?

8/11/2011 • page 20

- Turn and Talk teacher mathematics content and high leverage tasks
- This task has Multiple entry levels – higher level makes this rich and rigorous, and multiple learning styles

Slide 21

**Turn and Talk**

- Does this task have multiple entry points?
- How can it be differentiated?
- Will it help ELL students build understanding?

8/11/2011 • page 21

- What about the task we have done can provide all students with the opportunity to learn?
- Is this a differentiated task as it is? Self-differentiating? How could you differentiate further?

Slide 22

### Suggested support For ELLs

- Sentence frames
  - ☐ There are \_\_\_\_ counters **on** the paper.
  - ☐ There are \_\_\_\_ counter **off** the paper.
- English language proficiency level (expanding bridging)
  - ☐ \_\_\_\_ counters landed on the paper.
  - ☐ \_\_\_\_ counters landed off the paper.

8/11/2011 • page 22

- ESL (Students would think of the “land” as the ground not the verb land), “Landed” more difficult because past tense.
- English language proficiency levels: entering, beginning, developing, expanding and bridging
- This information should be coming from the ESL teacher as you work cooperatively to help students grow and learn.

Slide 23

$$8 + 4 = [ \quad ] + 5$$

8/11/2011 • page 23

- Think for a minute about your answer to this problem, and what students in 1-6 grades might think the answer is.
- What goes in the box? What might students say?
- Children’s Mathematics: Cognitively Guided Instruction (CGI), by *Carpenter, Fennema, Franke, Levi & Empson, 1999*

Slide 24

$$8 + 4 = [ \quad ] + 5$$

	Percent Responding with Answers			
Grade	7	12	17	12 & 17
1 <sup>st</sup> - 2 <sup>nd</sup>				
3 <sup>rd</sup> - 4 <sup>th</sup>				
5 <sup>th</sup> - 6 <sup>th</sup>				

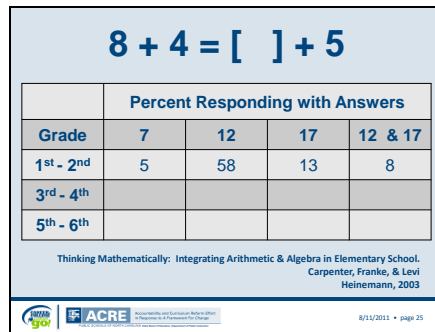
Thinking Mathematically: Integrating Arithmetic & Algebra in Elementary School.  
Carpenter, Franke, & Levi  
Heinemann, 2003

8/11/2011 • page 24

- Across the top you see the various answers students offered: 7, 12, 17, 12 & 17.
- How did they get each of these responses?

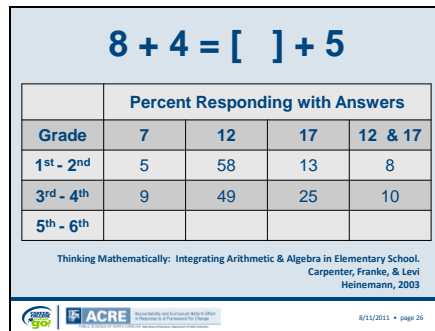


Slide 25



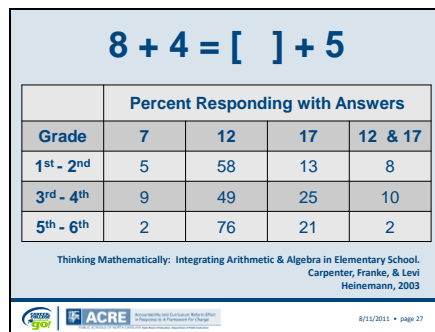
- We can see that 5 percent of 1-2 graders produced the correct answer.
- However, 58 percent thought the answer was 12.
- How did they get that?

Slide 26



- Now we look at 3-4. 2% more were right.
- Why do 12 % more students think 17 is correct?

Slide 27



- Now 5<sup>th</sup> -6<sup>th</sup> grades. The good news is that very few still think there are 2 answers. The bad news is that we are down to 2 % getting the right answer. Procedures memorized but not understood are getting in the way.
- Barbara Bissell used this in Charlotte as a benchmark assessment and only 25% of 3<sup>rd</sup> and 4<sup>th</sup> grade students got it right.
- Dr. Drew Polly at UNCC replicated this study for 6<sup>th</sup> grade, and none of the students got the correct the answer.

Slide 28

Operations and Algebraic Thinking
K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

**NOTE:** Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.  
(page 11)

Accountability and Curriculum Policy Office  
Department of Education  
www.doe.state.nj.gov
8/11/2011 • page 28

- This is the first standard we refer to in the power point. Review parts of the standards – **domain, bold heading of cluster and the standards**. Use vocabulary to get participants familiar with the new terms.
- Equality concepts are introduced in Kindergarten as students work with combining and separating situations as they decompose different amounts.
- Note about equations: While the standards include reading/writing equations in the standards, using and understanding the equal sign is not an explicit standard until First Grade. Equality is one of the mathematical properties that is fundamental in understanding mathematics, and begins in Kindergarten with the conceptual understanding before the introduction of symbols (+, -, =).
- Think of how our simple game On/Off addresses this concept. This standard calls on teachers to focus on this property as much as the 'answer' to 3+4 and 4+3.

Slide 29

Accountability and Curriculum Policy Office  
Department of Education  
www.doe.state.nj.gov
8/11/2011 • page 29

- Children can use counting boards and counters to tell a story in which addition and subtraction are represented.
- After students have been introduced to the equal sign, then a number sentence can be used to either represent a problem (find a number sentence that represents your story), or create the context for that problem (pick a number sentence and create a story).

Slide 30

**Operations and Algebraic Thinking** **1.OA**

**Work with addition and subtraction equations.**

1.OA.7 **Understand the meaning of the equal sign**, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false?  $6 = 6$ ,  $7 = 8 - 1$ ,  $5 + 2 = 2 + 5$ ,  $4 + 1 = 5 + 2$ .

**Think of  $8 + 4 = \underline{\quad} + 5$ .**

**Building on tasks like On and Off, students can think about how**

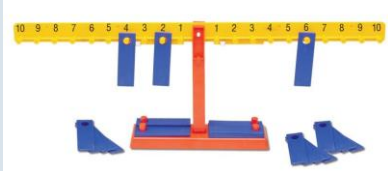
**$3 + 4 = 5 + 2$  or  $5 + 2 = 6 + 1$**

8/12/2011 • page 30

- The clearer higher standards in CCSS *explicitly* address equality in FIRST grade. Equality is one of the mathematical properties that is fundamental in understanding mathematics.

Slide 31

**Number Balance**



8/12/2011 • page 31

- It is important for children to develop the understanding of the concept of equality and how the equal sign is used to represent the relationship between one side of the equation and the other, versus the misconception that the equal sign means “answer comes next”.
- A number balance is an excellent tool to use to help children develop a sense of equality.

Slide 32

**Operations and Algebraic Thinking** **2.OA**

**Represent and solve problems involving addition and subtraction.**

2.OA.1. **Use addition and subtraction within 100** to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, **with unknowns in all positions**, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem

8/12/2011 • page 32

- We want to move beyond the “typical” word problem, and provide many types of problems in which children are using the concept of equality to find solutions.

Slide 33

### Grade Two

**Students use context problems to learn about a variety of problem structures.**

- Sneha has 9 apples. Kitty gives her 4 more. How many does she have?  $9 + 4 = \underline{\quad}$
- Sneha has some apples. Kitty gives her 4 more. Now she has 11 apples. How many did she have to begin with?  $\underline{\quad} + 4 = 11$

8/11/2011 • page 33

- Click to show each sample problem.
- CCSS stresses the importance of context problems in understanding mathematics. Different problem structures call on students to reason about quantities, operations, and reasonableness.
- These are based on the Cognitively Guided Instruction (CGI) problem types (Carpenter, et. al, 2002).

Slide 34

### Common Core Glossary

**Table 1. Common addition and subtraction situations**

	Result Unknown	Change Unknown	Start Unknown
<b>Add to</b>	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
<b>Take from</b>	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
<b>Put Together/ Take Apart</b>	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + \underline{\quad}, 5 = 5 + 0, 5 = 1 + 4, 5 = 4 + 1$

8/11/2011 • page 34

- This is a reference, and not meant to be read by the participants.
- This table identifies each of the different problem types and can be found in the Glossary on page 88 in the CCSS document. It is also located in the DPI Unpacking Document Appendix.

Slide 35

### Grade Three

- Understanding different problem structures moves students to multiplication and its relationship to division.
- Table 2 on page 89

8/11/2011 • page 35

Look at Standards to find all types.

Slide 36

Common Core Glossary			
Table 2. Common multiplication and division situations			
	Unknown Product (How many in each group?) Example: $3 \times 4 = ?$	Group Size Unknown (How many groups?) Example: $12 \div 3 = ?$	Number of Groups Unknown (How many groups?) Example: $12 \div ? = 3$
Class Groups	There are 3 bags with 4 plants in each bag. How many plants are there in all? Measurement example: You have 12 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 12 plants are shared equally into 3 bags, then how many plants will be in each bag? Measurement example: You have 12 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 12 plants are to be packed 4 to a bag, then how many bags are needed? Measurement example: You have 12 inches of string, which you will cut into pieces that are 4 inches long. How many pieces of string will you have?
Area/Area	There are 3 rows of apples with 4 apples in each row. How many apples are there? Area example: A rectangle has a length of 3 cm and a width of 4 cm. What is its area?	If 12 apples are arranged into 3 equal rows, how many apples will be in each row? Area example: A rectangle has a length of 3 cm and a width of 4 cm. What is its area?	If 12 apples are arranged into equal rows of 4 apples, how many rows will there be? Area example: A rectangle has a length of 3 cm and a width of 4 cm. What is its area?
Compare	A blue hat costs \$4. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example: A rubber band is stretched to be 10 cm long and that is 3 times as long as the rubber band at rest. How long is the rubber band at rest?	A red hat costs \$4 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? Measurement example: A rubber band is stretched to be 10 cm long and that is 3 times as long as the rubber band at rest. How long is the rubber band at rest?	A red hat costs \$4 and a blue hat costs \$2. How many times as much does the red hat cost as the blue hat? Measurement example: A rubber band was 10 cm long at rest. Now it is stretched to be 30 cm long. How many times as long is the rubber band now as it was at rest?
General	$a \times b = ?$	$a \times ? = c$ and $p \div a = ?$	$? \times b = c$ and $p \div b = ?$

- This is a reference; this is not meant to be read! Participants refer to their copy. Glossary page 89.
- Make problem types explicit.

Slide 37


### Third Grade Critical Area

(1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; **multiplication is finding an unknown product, and division is finding an unknown factor in these situations.** For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

- The common Core identifies 3 – 5 Critical Areas of Focus for each grade level. These appear at the beginning of each grade level.
- This page is a reference to the standards, everything written in black in this power point comes directly from the standards and participants should refer to their standards this is only to be a refer page for them – NOT meant for them to read.

Slide 38

There are 4 space ships.  
Each spaceship has 3  
durdles (legs). There are  
12 durdles all together.



Write three riddles for this story with three different answers.  
(Put an unknown in each position. Table 2, p. 89)



- Here we have an example of a multiplication situation. There are several operational problems that could be written for this situation, as outlined in the Critical Focus 1 for 3<sup>rd</sup> grade.
- Write three riddles for this situation with the unknown in each of 3 places. Pause for participants to write riddles.

Slide 39

**Operations and Algebraic Thinking** 4.OA

**Work with addition and subtraction equations.**

**4.OA.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, **including problems in which remainders must be interpreted.** Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.



  Assessment and Curriculum Review Office  
Improving the Quality of Florida's Schools 8/11/2011 • page 39

Remainders must be interpreted, clearer

Slide 40

**$48 \div 5 =$**

a) 10  
b) 9 r 3  
c)  $9 \frac{3}{5}$   
d) 9.60

  Assessment and Curriculum Review Office  
Improving the Quality of Florida's Schools 8/11/2011 • page 40

• 48 divided by 5. Which answer is correct?

Turn and talk:

Problem with Context. How many vans will you need to take 48 people to the park if 10 people can ride in a van?

Turn and talk. Make up a problem such that each of these answers is correct.

10 -----

9 r 3 48 balloons divided among 5 kids

•  $\frac{3}{5}$  Brownies divided

9.60 money \$48.00 mowing 5 lawns, how much per lawn. or time(seconds)

Slide 41



**Operations and Algebraic Thinking** 5.OA

**Write and interpret numerical expressions.**

**5.OA.2** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as  $2 \times (8 + 7)$ .* Recognize that  $3 \times (18932 + 921)$  is three times as large as  $18932 + 921$ , without having to calculate the indicated sum or product.

**Which of the following could be an estimate for the number represented by this expression:  $3(18933 + 921)$**

a) 19854  
b) 60000  
c) 319854  
d) 30,000

  Assessment and Curriculum Review Office  
Improving the Quality of Florida's Schools 8/11/2011 • page 41

This example came directly from the standard. Don't overlook examples.

Slide 42

**Operations and Algebraic Thinking** 5.OA

**Write and interpret numerical expressions.**

**5.OA.2** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as  $2 \times (8 + 7)$ . Recognize that  $3 \times (18932 + 921)$  is three times as large as  $18932 + 921$ , without having to calculate the indicated sum or product.*

Which of the following could be an estimate for the number represented by this expression:  $3(18933 + 921)$

- a) 19854
- b) 60000
- c) 319854
- d) 30,000

8/12/2011 • page 42

Be sure to look at examples to build understanding and expectations of the standard.

Slide 43

**Exploring Algebra**

- **Work with a partner.**
- **Look at one or more grade levels of the Operations and Algebraic Thinking Standards.**

8/12/2011 • page 43

- Now it's your turn to dig deeply into the standards.
- We have seen an example of early algebra for each grade level, but there is more.
- Put on your algebra glasses and look for the early algebra in some of the skills we have always taught.
- Properties – rules by which number operate

Slide 44

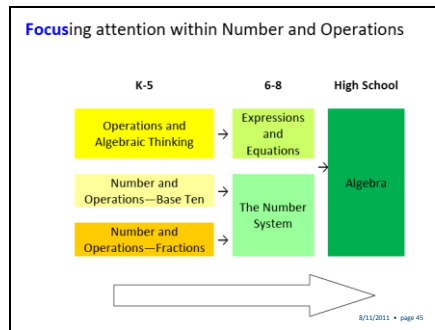
**"Arithmetic is a rehearsal for algebra."**

Bill McCullam, CCSS Mathematics Author

8/12/2011 • page 44

Be intentional about algebraic thinking with children

Slide 45



Progression of how algebra builds K-12

Slide 46

**Old Boxes**

- People are the next step
- If people just swap out the old standards and put the new CCSS in the old boxes
  - into old systems and procedures
  - into the old relationships
  - Into old instructional materials formats
  - Into old assessment tools,
- Then nothing will change, and perhaps nothing will

Phil Daro, NCCTM 2010

8/12/2011 • page 46

Phil Daro

Slide 47

**Time to Reflect**

Summary	

8/12/2011 • page 47

- Explain.....Reflect on what you have heard. In the upper left hand corner.... What are some ah-ha's, things you want to remember?
  - Mathematical content
  - Teacher evaluation (what do principals need to see when they walk in your room?)
  - Professional Development

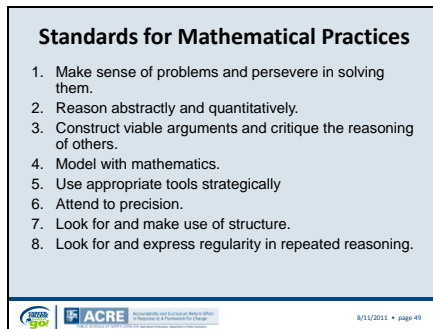


Slide 48



Share cartoon

Slide 49

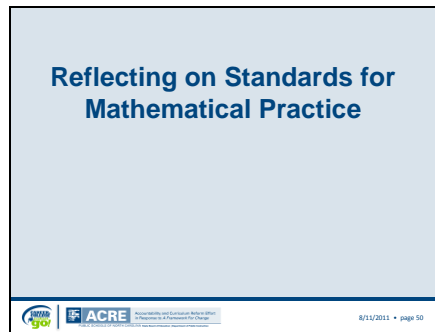


Survey group for familiarity.

Refer task completed in algebra

- Find this in your standards
- Carry across all grade levels
- Describe habits of a mathematically expert student
- The Common Core proposes a set of Mathematical Practices that all teachers should develop in their students. These practices are similar to the mathematical processes that NCTM addresses in the Process Standards in *Principles and Standards for School Mathematics*.
- These can be implemented **now** even before content goes into effect.

Slide 50



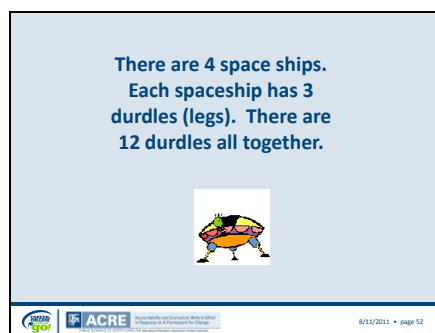
Find Handout of standards work in groups to complete the handout - example on next slide.

Slide 51

Teacher Actions (Cause)	Student Practices (Effect)
1. Provide time for and facilitates the discussion of problem solution.	<p>1. Make sense of problems and persevere in solving them.</p> <p>Do Students:</p> <ul style="list-style-type: none"> <li>• Use multiple representations (verbal descriptions, symbolic, tables, graphs, etc.)?</li> <li>• Explain to themselves the meaning of the problem.</li> <li>• Check their answers using different methods?</li> <li>• Continually ask, "Does this make sense?"</li> <li>• Understand the approaches of others and identify correspondences between different approaches?</li> </ul>

- Using the practices complete this handout (participants reflect on 4 of the 8 practices not time to do all, just giving participants an idea of how this could be addressed in their region)
- Participants complete four tables like this.
- What do teachers do in their classrooms to cause these effects on students?
- Do a few together to get them started.
- Provide time for and facilitates the discussion of problem solution.
- Teaching Evaluation Standards

Slide 52



Think about what practices you used during this task.  
What were the students doing?  
What was the teacher doing?

Slide 53



Timer if needed

Slide 54

**1. Make sense of problems and persevere in solving them.**

**DO STUDENTS:**

- Use multiple representations (verbal descriptions, symbolic, tables, graphs, etc.)?
- Check their answers using different methods?
- Continually ask “Does this make sense?”
- Understand the approaches of others and identify correspondences between different approaches?

The slide footer includes logos for 'middle school' and 'ACRE' (Assessment and Curriculum Review Office), the text 'Assessability and Curriculum Review Office', 'Assessing and Improving the Quality of Instruction', and the date '8/11/2011 • page 54'.

Participants share  
What do teachers do in their classrooms to cause these effects on students?

Slide 55

**1. Make sense of problems and persevere in solving them.**

The slide footer includes logos for 'middle school' and 'ACRE' (Assessment and Curriculum Review Office), the text 'Assessability and Curriculum Review Office', 'Assessing and Improving the Quality of Instruction', and the date '8/11/2011 • page 55'.

1. Make Sense of problems and persevere in solving them.  
Share any thoughts that were not mentioned:  
The mathematically proficient student will:



- Explain the meaning of the problem.
- Analyze givens.
- Make conjectures.
- Plan a solution path.
- Consider analogous problems.
- Look for a simpler problem.
- Monitor and evaluate progress.
- Use concrete objects or pictures to conceptualize the problem.
- Represent the solution using equations, tables, graphs, and data.
- Solve the problem using different methods.
- Explain the relationship between different solutions.

Slide 56

**2. Reason abstractly and quantitatively.**

**DO STUDENTS:**

- Make sense of quantities and their relationships in problem situations?
- Decontextualize a problem?
- Contextualize a problem?
- Create a coherent representation of the problem, consider the units involved, and attend to the meaning of quantities

  ACRE Assessment and Evaluation of Mathematics Education  
Improving the Quality of Mathematics Education

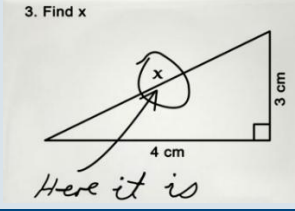
8/12/2011 • page 56

Participants do not address this standard during this session.  
 Will address this practice later when doing mathematical tasks but include comments from next slide



Slide 57

**2. Reason abstractly and quantitatively.**

3. Find  $x$



*Here it is*

  ACRE Assessment and Evaluation of Mathematics Education  
Improving the Quality of Mathematics Education

8/12/2011 • page 57

2. Reason abstractly and quantitatively.  
 The mathematically proficient student will:



- Demonstrate the ability to decontextualize a problem symbolically.
- Manipulate symbols.
- Contextualize symbols in light of the original problem.
- Create a coherent representation of the problem.
- Attend to the meaning of the quantities involved.

Slide 58

**3. Construct viable arguments and critique the reasoning of others.**

**DO STUDENTS:**

- Make conjectures and build a logical progression of statements to explore the truth of their conjectures?
- Analyze situations and recognize and use counter examples?
- Justify their conclusions, communicate them to others, and respond to arguments of others?
- Hear or read arguments of others and decide whether they make sense, and ask useful questions to clarify or improve the argument?



  Assessment and Evaluation Policy Office  
University of Connecticut 8/11/2011 • page 58

Let participants share: What do teachers do in their classrooms to cause these effects on students?

Slide 59

**3. Construct viable arguments and critique the reasoning of others.**

- **7.8 x .98**
- **45.1 x 1.05**
- **0.52 x 15.6**

  Assessment and Evaluation Policy Office  
University of Connecticut 8/11/2011 • page 59

3. Construct viable arguments and critique the reasoning of others.

Share any thoughts participants don't address as needed:

The mathematically proficient student will:



- Understand and use stated assumptions, definitions, and previously results to construct
- Arguments.
- Make conjectures and build a logical argument.
- Can break problems in cases.
- Can recognize and use counter arguments.
- Justify their arguments.
- Communicate their arguments to others.
- Reason inductively.
- Compare and evaluate the effectiveness of two arguments or solutions.
- Distinguish between correct and flawed logic.
- Use concrete referents such as objects, drawings diagrams or actions where appropriate
- For making age appropriate arguments.
- Listen to the arguments of others and ask appropriate questions to help decide if the
- Arguments make sense.

Slide 60

**4. Model with mathematics.**

**DO STUDENTS:**

- Apply the mathematics they know to solve problems in everyday life?
- Apply what they know and make assumptions and approximations to simplify a complicated situation as an initial approach?
- Identify important quantities in a practical situation?
- Analyze relationships mathematically to draw conclusions?
- Interpret their mathematical results in the context of the situation and reflect on whether the results make sense?

  ACRE Assessment and Continuous Improvement  
Assessment and Continuous Improvement



8/12/2011 • page 60

Participants did not do this standard.  
Will address this practice later when doing mathematical tasks put may include comments from next slide.

Slide 61

**4. Model with mathematics.**

$$\frac{1}{2} + \frac{2}{3} =$$

  ACRE Assessment and Continuous Improvement  
Assessment and Continuous Improvement

8/12/2011 • page 61



4. Model with mathematics.  
The mathematically proficient student can:  
Apply the mathematics that they know to solve problems arising in everyday life.  
Interpret their results in the context of the problem and reflect on whether the results make sense.

Slide 62

**5. Use appropriate tools strategically.**

**DO STUDENTS:**

- Consider the available tools when solving mathematical problems?
- Know the tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful?
- Identify relevant external mathematical resources and use them to pose or solve problems?
- Use technological tools to explore and deepen their understanding of concepts?

  ACRE Assessment and Continuous Improvement  
Assessment and Continuous Improvement

8/12/2011 • page 62

Let participants share: What do teachers do in their classrooms to cause these effects on students?

Slide 63



## 5. Use appropriate tools strategically.

The mathematically proficient student will:

- Select from available tools those that are appropriate for solving the problem at hand.
- Be sufficiently familiar with both traditional mathematical tools (calculators, compass, Ruler, protractors, concrete models, etc.) and modern mathematical tools (graphing Calculators, CAS, dynamic geometry software, statistical packages, etc.) so that they can
- make sound decisions about when each of these tools might be helpful and their
- Limitations as problem solving tools.
- Be able to identify relevant external mathematical resources, such as digital content
- Located on websites, tutorials, CASs and use them to pose or solve problems.
- Be able to use technological tools to explore and deepen their understanding of concepts.

Slide 64

## 6. Attend to precision.

**DO STUDENTS:**

- Communicate precisely to others?
- Use clear definitions?
- Use the equal sign consistently and appropriately?
- Calculate accurately and efficiently?

ACRE

8/11/2011 • page 64

Let participants share: What do teachers do in their classrooms to cause these effects on students?

## 6. Attend to precision.

The mathematically proficient student will:  
Be able to communicate precisely with others.



Be able to measure using various measuring tools accurately and be able to state the correct units of measure for the stated problem.

Make explicit use of definitions, axioms, theorems, etc.

## 7. Look for and make use of structure.

**DO STUDENTS:**

- Look closely to determine a pattern or structure?
- Utilize properties?
- Decompose and recombine numbers and expressions?

  Improving and Sustaining Mathematical Thinking & Performance Through 21st Century Learning 8/11/2011 • page 66

Will address this practice later when doing mathematical tasks put may include comments from next slide

Focus on properties. Numbers behave the same way K-12. Only 9 properties, we use each property but don't necessary expect students to call them by name.

7. Look for and make use of structure.

$3 \times 6$   
 $3 \times 60$   
 $3 \times 62$   
 $3 \times 70$   
 $3 \times 68$

Where's the math?

The mathematically proficient student will:  
Develop the ability to discern the patterns or structures present in mathematical problems or contexts.





Slide 68

**8. Look for and express regularity in repeated reasoning.**

**DO STUDENTS:**

- Notice if calculations are repeated, and look both for general methods and for shortcuts?
- Maintain oversight of the process, while attending to the details?
- Continually evaluate the reasonableness of their intermediate result?

  Accountability and Curriculum Review Office  
Department of Education  
8/11/2011 • page 68



Participants did not do this standard. Will address this practice later when doing mathematical tasks but may include comments from next slide.

Slide 69

**8. Look for and express regularity in repeated reasoning.**

If  $24 \div 6$  means how many groups of 6 can I make from 24,  
how can I use that understanding to determine

$6 \div \frac{1}{2}$

  Accountability and Curriculum Review Office  
Department of Education  
8/11/2011 • page 69

8. Look for and express regularity in repeated reasoning.



The mathematically proficient student will:

- Develop the ability to notice the regularity of answers and results from repeated
- mathematical procedures.
- Look for general methods for solving related problems.

Slide 70

**Standards for Mathematical Practices**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.


  Accountability and Curriculum Review Office  
Department of Education  
8/11/2011 • page 70

- Carry across all grade levels
- Describe habits of a mathematically expert student
- The Common Core proposes a set of Mathematical Practices that all teachers should develop in their students. These practices are similar to the mathematical processes that NCTM addresses in the Process Standards in *Principles and Standards for School Mathematics*.
- These can implemented between now and time content goes into effect.

Slide 71

**Mathematical practices describe the habits of mind of mathematically proficient students...**

- **Who is doing the talking?**
- **Who is doing the math?**


 8/11/2011 • page 71

- Note that, the standards for Mathematical practice describe the habits of mathematically proficient students...
- So how do we help kids develop these behaviors?
- As Cathy Seely said: In your math class, who is doing the talking? Who is doing the math?


Slide 72

**Students View Themselves as Mathematicians**

*Students as Mathematicians*

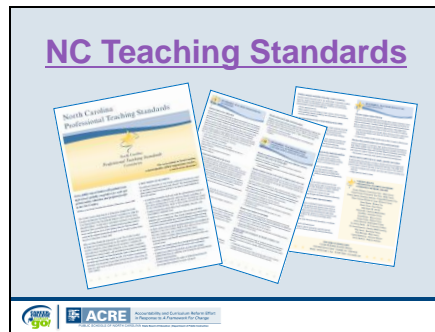


<http://www.curriculum.org/secretariat/coplanning/>

 8/11/2011 • page 72

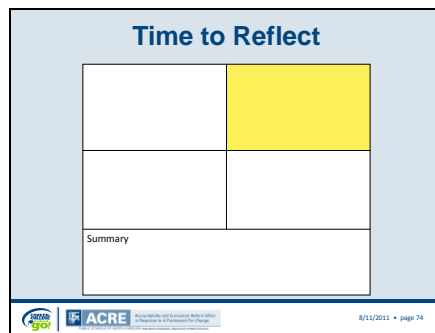
- Hyperlink - video of students doing mathematics.
- Using information tech standards by using this video. Tech should be embedded in everything we do.
- INFORMATION and TECHNOLOGY ESSENTIAL STANDARDS**
- SI: Sources of Information**
- Evaluate sources
  - Evaluate information
  - Classify sources of information
- IN: Informational Text**
- Compare facts
  - Summarize
  - Use technology tools to access information
- TT: Technology as a Tool**
- Use a technology tool to organize data
  - Use a technology tool to present data
- RP: Research Process**
- Apply the process
  - Implement a project-based activity that uses research
  - Use a research process as part of a collaborative activity
- SE: Safety and Ethical Issues**
- Identify responsible use and care of technology
  - Show respect for the work of others
  - Analyze ethical behavior
- [http://it.ncwiseowl.org/curriculum\\_instruction/eStandards/](http://it.ncwiseowl.org/curriculum_instruction/eStandards/)
- <http://www.ncpublicschools.org/acre/standards/new-standards/>

Slide 73



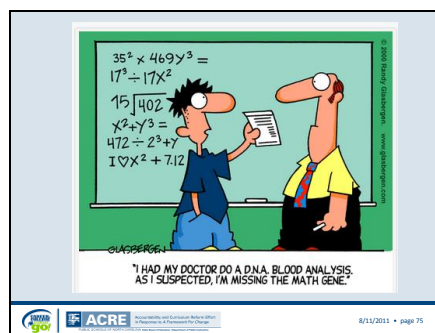
- According to the North Carolina Professional Teaching Standards, Teachers Are Challenged To: Collaborate with their colleagues and use a variety of data sources for short and long range planning based on the North Carolina Standard Course of Study.
- Engage students in the learning process and understand that instructional plans must be constantly monitored and modified to enhance learning.
- Understand how students learn and make the curriculum responsive to cultural diversity and to individual learning needs.

Slide 74



Time to reflect

Slide 75





Share cartoon. How many of you have heard "Don't ask me. I don't do math."? How about "I don't do read."? While we may laugh about our lack of math knowledge, it's a national concern.

Slide 76

**Standards for Mathematical Practices**



1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

  Accountability and Curriculum Review Office  
Division of Assessment and Research  
New York State Education Department 8/11/2011 • page 76

- Carry across all grade levels
- Describe habits of a mathematically expert student
- The Common Core proposes a set of Mathematical Practices that all teachers should develop in their students. These practices are similar to the mathematical processes that NCTM addresses in the Process Standards in *Principles and Standards for School Mathematics*.
- These can implemented between now and time content goes into effect.

Slide 77

**Composing and Decomposing Number**

  Accountability and Curriculum Review Office  
Division of Assessment and Research  
New York State Education Department 8/11/2011 • page 77

Decomposing is not about roadkill.

Slide 78



**Counting and Cardinality** **K.CC**

Count to tell the number of objects.

**K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.**

- a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
- b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
- c. Understand that each successive number name refers to a quantity that is one larger.

**K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.**

  Accountability and Curriculum Review Office  
Division of Assessment and Research  
New York State Education Department 8/11/2011 • page 78

- This is the second cluster under Counting and Cardinality in Kindergarten, K.CC. Counting is one of five Domains in Kindergarten.
- Learning to count is extremely complex. For that reason, many of the counting benchmarks are very clearly articulated. Give example of child giving you the fourth one, not four objects. This is Higher than just learning to sing the 1-2-3-4- song.
- 4.a One – to – one remind folks how young students just count and count and count, saying the words unrelated to what they are touching.
- 4.B Counting tells how many, regardless of how the objects are arranged - **conservation**, a small pile and a long line are still the same

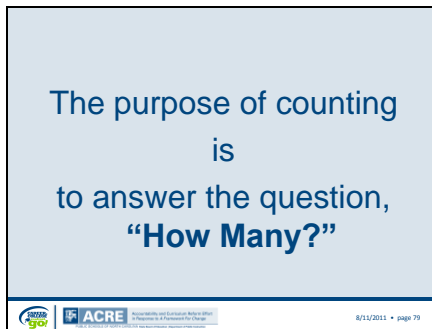
quantity

5. Clearer—read actual standard for 5

- May be able to sing Feliz Navidad and not speak French.

Slide 79

The purpose of counting  
is  
to answer the question,  
**“How Many?”**

The slide has a light blue background with the text centered. At the bottom, there is a footer with logos for 'eSOL' and 'ACRE' (Arizona Center for Educational Reform), along with the date '8/11/2011' and page number 'page 79'.

It is important to pose counting opportunities to students in a problem-to-be-solved manner (How many do you think there might be? Let's count to see. How many were there? Were you surprised? Do you think this set will have more than this one? Why do you think so?) vs. a task to be completed (Count this group of objects and write the numeral.).

Slide 80

**One-to-One Correspondence**

The slide features a photograph of a wooden tray divided into three sections. The left section contains five blue pom-poms. The middle section contains five red pom-poms. The right section contains a grid with five red pom-poms placed in the first five rows. Above the tray, there are two small bowls, one with blue pom-poms and one with red pom-poms. The title 'One-to-One Correspondence' is written in blue above the image. The footer at the bottom includes the 'eSOL' and 'ACRE' logos, the date '8/11/2011', and page number 'page 80'.

- 1-to-1 correspondence refers to correspondence between two collections if every member of each collection is paired with exactly one member of the other collection and no members of either collection is unpaired or is paired with more than one number (NRC, 2009, p. 354).

Slide 81



- One to one tagging refers to the child's ability to count and point to objects simultaneously. The child must be able to have 1-to-1 correspondence as s/he "tags" each item with one counting word.
- Also known as "object counting" (NRC, 2009) and "synchronic counting" (Fosnot & Dolk, 2001)

Slide 82



- Cardinality: To know the number you say tells the quantity you have and that the number you end on when counting represents the entire amount involves cardinality- the idea that number means "amount". (Fosnot & Dolk, 2001, p. 33)
- Without this understanding, counting is little more than a singsong activity to young children, a rote action they perform when adults ask them to see how many of something there are (Fosnot & Dolk, 2001, p. 33)

Slide 83



- Keeping track is a method of counting that is used to count each item once and only once when determining how many.
- There are many ways to keep track: (Kathy Richardson, 2002)
- Lines Up: The child lines up the objects before counting them. This method is usually used b/c the child has been shown this way to keep track. Lining up the counters before counting may help children get right answers, but it also limits their flexibility and interferes with the development of more efficient strategies.
- Looks: The child tries to count without touching the counters. Sometimes children count by looking b/c they don't yet realize touching the counters helps... others are confident and want to show that they can count with their eyes.

- Points:** The child points at the objects without moving them.

On one hand, a child who counts without moving them may not have a way of keeping track. On the other hand, it could be a sign that a child doesn't need to move them and can keep track without moving anything.

- Moves:** The child moves each counter as he or she counts it.

This is the most common way for young children to keep track of counting objects.

- Keeping track is a strong indicator of whether or not a child understands and can use the important ideas of counting.

- A child keeps track when s/he understands that a set of objects has a certain amount- and that when counted over and over again, the set remains that amount.

- Many times, a young counter is able to keep track of counting a small amount of objects (7) but is unable to keep track of larger amounts (14). Thus, a child is not either "able to keep track" or "not able to keep track", but is able to keep track to a certain point.

- The problem with counting objects that are stationary is that you can't tell if students are aware of the importance of 'keeping track' so they know how many. Workbook pages don't work. So, real, concrete experiences are essential.

Slide 84

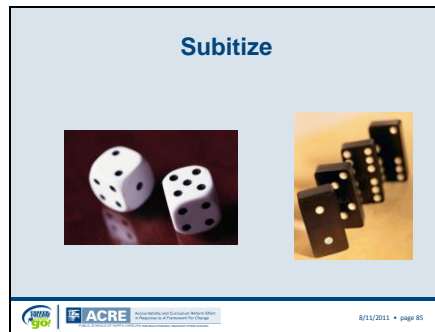


- Also known as "Hierarchical Inclusion" (Kamii, 1985; Piaget 1965)

- Inclusion is the idea that numbers build by exactly one each time and that they nest within each other by this amount. For example, within a set of 4 objects are a set of three objects; If I have four objects and want to have 5 objects, I add one more- knowing that four is part of 5 (rather than removing all 4 objects and starting over to make a new set of 5). (Fosnot & Dolk, 2001, p. 36)

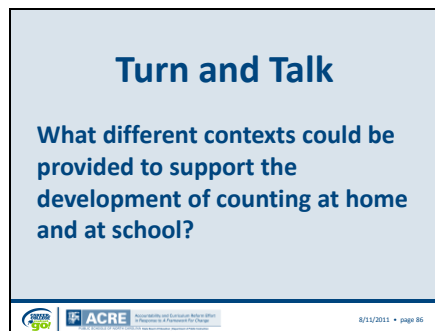
- This concept is critical for the later development the big ideas of compensation and part/whole relationships.

Slide 85



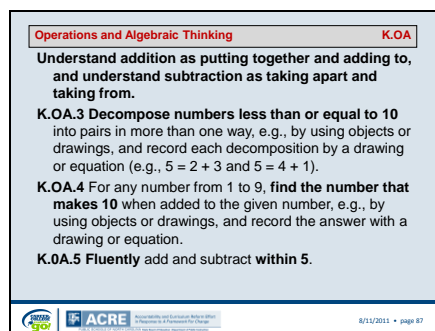
- A child is able to subitize when small amounts (up to 4-5) can be seen as a whole. Sh/e is able to know the total amount instantly without counting. (ex: pips on dominoes, die)
- This ability is an important skill that moves children beyond “counting all”, towards “counting on”, and ultimately to “decompose/compose number” to make tens-our ultimate goal

Slide 86



Counting is complex!  
Turn and Talk, and don't forget to include on your list things the kids can do at home.

Slide 87



- This cluster focuses on the part-whole relationships of numbers 1-10. Children need to have inclusion (ability to see sets within the larger set) before they begin internalizing the various parts of the whole.
- While students use objects, drawings, etc. to show the various parts of numbers 1-10, Standard K.OA.5 asks that kindergarteners have internalized the combinations of 2, 3, 4 and 5. This means that they are accurate (correct answer), efficient (reasonable amount of steps), and flexible (use strategies).
- Students develop fluency by working with many different kinds of concrete materials (such as cubes, chips, and buttons) over an extended amount of time.
- The Standard K.OA.5 is not about looking at




a flashcard with  $3 + 2 = ?$  and asking the child to give you the answer quickly (Remember- the equal sign is not introduced until First Grade), but rather, having the child count out 5 counters, you hide 3, show the remaining 2, ask the child “how many are hiding?”, and having the child be fluent with the answer (Kathy Richardson, Hiding Assessment).

- Traditional flash cards or timed tests have not been proven as effective instructional strategies for developing fluency. Rather, numerous experiences with breaking apart actual sets of objects help children internalize parts of number- thus becoming fluent.

Slide 88

### Let's Make Tens

•Get a partner, some color tiles, and a ten frame.


8/12/2011 • page 88

- Let's use a ten frame.

- Ten frames can be placed vertically or horizontally (we want children to see number in a variety of ways= flexible); however, most often, ten frames are used horizontally, with chips placed moving top to bottom, left to right. Some kindergarteners are still pre-readers, and need experience moving in this manner- with a “return sweep”.



- “Show me 4.”

- “Show me 12.” There is not enough space... the extra two are off of the ten frame, thus 12 is “ten and some more”.

Slide 89

### Let's Make Tens

- Partner 1 draws a number card and fills in the ten frame.
- Partner 2 tells how many are needed to make ten.
- Clear the frame.
- Now the other partner draws a number and fills the frame.

  Accountability and Curriculum Review Office  
Improvement in Assessment for Change 8/12/2011 • page 89



Directions to task

Slide 90

### Turn and Talk

What did you notice...

- about ways to make ten?
- about learning basic facts?
- about the importance of visual and kinesthetic representation?



  Accountability and Curriculum Review Office  
Improvement in Assessment for Change 8/12/2011 • page 90

Turn and Talk at your table.

Slide 91

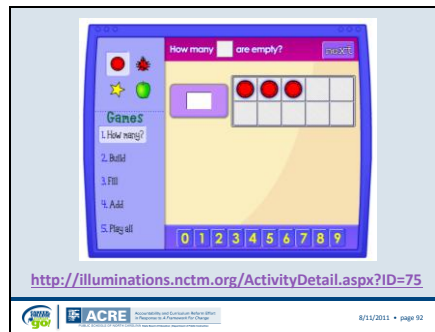
### Turn and Talk

- Does this task have multiple entry points?
- How can it be differentiated?
- Will it help ELL students build understanding?

  Accountability and Curriculum Review Office  
Improvement in Assessment for Change 8/12/2011 • page 91

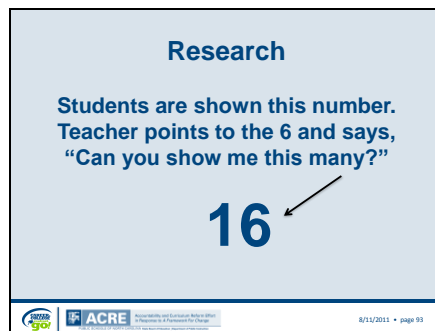
What about the task we have done can provide all students with the opportunity to learn?

Slide 92



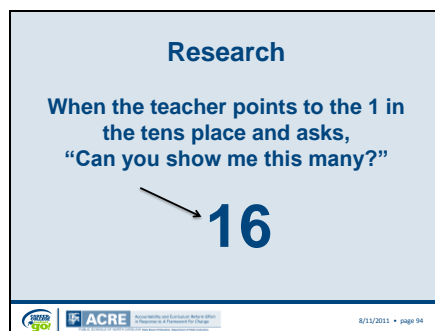
Hyperlink to model task  
Tech standard

Slide 93



Constance Kamii has done extensive research on how young children learn mathematics. Here is a task frequently used to assess understanding of tens and ones. Students usually are successful, and count out 6 blocks.

Slide 94





Kamii found that essentially no first graders could correctly complete this task. Rather than counting out 10 blocks, they only counted 1 block.

Slide 95

**Research**

By third grade nearly half the students still do not 'get' this concept.

**16**

  Assessment and Curriculum Review Office  
Department of Education, State of Michigan 8/11/2011 • page 95

Many 3-5 graders **still** do not give the correct answer.



Slide 96

**More research - It gets worse!**

A number contains 18 tens,  
2 hundreds, and 4 ones.  
What is that number?

**1824      2824**

**218.4      384**

  Assessment and Curriculum Review Office  
Department of Education, State of Michigan 8/11/2011 • page 96

- Grayson Wheatly's research with 5,000 middle-school students were given the following task:
- Some students gave this answer.
- Others knew the tens had to be in the middle, so.....
- Many gave this answer, knowing about decimals, and that you could only have 3 digits if a number was in the hundreds.
- Around 50% of the middle school students gave the correct answer.

Slide 97

**Common Core State Standards begin to specifically address this misunderstanding in Kindergarten and First Grade.**

  Assessment and Curriculum Review Office  
Department of Education, State of Michigan 8/11/2011 • page 97


Misunderstandings must be addressed

Slide 98

**Number and Operations in Base Ten** **K.NBT**

Work with numbers 11–19 to gain foundations for place value.

**K.NBT.1** Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g.,  $18 = 10 + 8$ ); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.



8/11/2011 • page 98

- It is very important to note that this objective does not ask kindergarten students to unitize a ten. Rather, all are kept as ones. It is not until First grade 1.NBT.2.a: 10 can be thought of as a bundle of ten ones- called a “ten”. Thus all examples and experiences around this standard need to keep all counts as “ones”. Cubes, counters, ten frames are good tools to use for this,
- “Show me 12” allows children to see all 10 counters on the ten frame and some left-over to the side. They can still count each individually to determine how many.

Slide 99

**Number and Operations in Base Ten** **1.NBT**

Understand place value.

**1.NBT.2** Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

- 10 can be thought of as a bundle of ten ones — called a “ten.”
- The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
- The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

8/11/2011 • page 99

- In this First Grade cluster, students are asked to unitize- to think of 10 ones as 1 ten. This is a complex concept and needs numerous experiences for children to make that leap.
- They also begin to see a ten as part of a larger number (such as 30).
- Rather than “telling” children, “There are three tens in 30.”, children must use numerous materials to discover that there is enough to make a ten in 30- in fact- there’s enough to make 3 tens and not have any left over- and make this connection that the 3 and the 0 in their bundles are represented in each of the digits in the number, “30”.

Slide 100

**Number and Operations in Base Ten** **1.NBT**

Understand place value.

**1.NBT.3** Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols  $>$ ,  $=$ , and  $<$ .

8/11/2011 • page 100

- Fewer, Clearer, Higher, and a coherent progression.
- Understanding symbols are important. The equal sign does not mean, “The answer comes next” but rather states “is the same amount/quantity as” to signify a relationship between the left side and right side of the equal sign.
- In addition, the  $>$  and  $<$  symbols are not simply alligators which eat a bigger number, but represent the understanding of “more than” and “less than”.

## Slide 101

**Number and Operations in Base Ten**
**1.NBT**

Use place value understanding and properties of operations to add and subtract.

**1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.**

**1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.**

**1.NBT.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.**

8/11/2011 • page 101

- This standard does not expect first graders to solve a 2-digit x 1-digit problem using the “regrouping” algorithm.
- Instead, the focus of these standards ask students to compose and decompose numbers to make tens. All standards mentioned here are based on the benchmark number 10.
- Thus, the problem  $36 + 8$  is not solved by lining up this problem in a vertical format and adding the ones side, carrying the “one” and adding the tens side.
- Instead, the problem  $36 + 8$  is solved by making tens:
- Decompose 8 into 4 and 4. Add the four to 36 to make 40. Then add the remaining 4 to make 44.
- Decompose 36 into 34 and 2. Add 2 to 8 making a ten. 34 and ten more is 44.

## Slide 102

**Ten Frames for  
Addition and Subtraction**

**Using the filled ten frames and  
the partially filled ones, create  
the number 45. Now subtract 20.**

**Brainstorm ways to use the ten  
frames to create tasks for first  
graders.**

8/11/2011 • page 102



- Find the cluster about using place value to add and subtract, p. 15. When the standards are this clear, you can create tasks more easily.
- Try this first task, and be sure to think like a first grader.
- Then, brainstorm with your group tasks you can give students to build understanding.

Slide 103

**Turn and Talk**

**How does this cluster build understanding of place value?**

**How is this different from the way we have traditionally taught place value?**

  Accountability and Curriculum Review Office  
Improvement in Assessment for Change 8/11/2011 • page 103



Newer, Better, Different

Slide 104

**$35 + 23$**

Using decomposing of number and the associative property, second graders may decide that since  $35 = 30 + 5$  and  $23 = 20 + 3$ ,

they can add  $20 + 30 = 50$  and  $5 + 3 = 8$ , then add  $50 + 8 = 58$

  Accountability and Curriculum Review Office  
Improvement in Assessment for Change 8/11/2011 • page 104



By second grade students add and subtract using their understanding of both the properties of mathematics and their understanding of place value.

Slide 105

**$35 + 23$**

**Decompose only the 23**

$35 + 20 = 55$   
then  
 $55 + 3 = 58$

  Accountability and Curriculum Review Office  
Improvement in Assessment for Change 8/11/2011 • page 105

While some refer to these as ‘invented strategies’ they are really methods commonly used when number composition and place value are understood.

Slide 106

**The distributive property can and should be used to teach multiplication.**

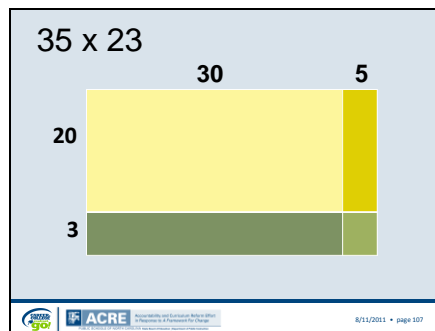
**35 X 23**

Let's use an area model.

8/11/2011 • page 106

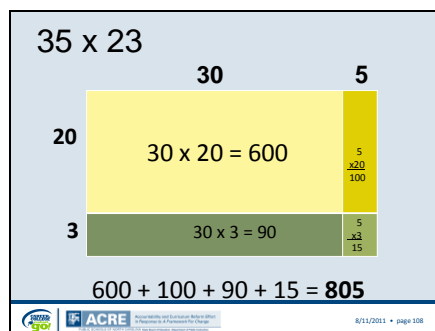
How does distributive property help teach multiplication.  
Let's look at a physical model on the next slide.

Slide 107



Here we have a rectangle that is 35 by 23 units in size.  
35 has been decomposed into 30 + 5, and 23 into 20 + 3.  
We have created 4 rectangular areas.

Slide 108





When we find the area of each by multiplying, we then add all the areas to get the total.



Slide 109

$$\begin{array}{r} 35 \\ \times 23 \\ \hline 15 \\ 90 \\ \hline 100 \\ 600 \\ \hline 805 \end{array}$$



  Accountability and Curriculum Review Office  
Preparation for Assessment for Change  
8/11/2011 • page 109

If we understand the place value ideas of the involved numbers, we can write each of the products without the model, resulting in what some call the partial product method.

Slide 110

**“With each extension of number, the meanings of addition, subtraction, multiplication and division are extended.”**

Common Core State Standards



  Accountability and Curriculum Review Office  
Preparation for Assessment for Change  
8/11/2011 • page 110

- The importance of identifying and stressing the mathematical properties in elementary school cannot be over emphasized.
- The properties are the rules by which numbers operate.
- This slide and the next are a preface to high school standards.

Slide 111

**“In each new number system- integers, rational numbers, real numbers, and complex numbers - the four operations remain the same in two important ways: they follow the same properties and their meanings are consistent with their previous meanings.”**

Common Core State Standards



  Accountability and Curriculum Review Office  
Preparation for Assessment for Change  
8/11/2011 • page 111

This quote came from the CCSS high School section

Slide 112

**“Arithmetic is a rehearsal  
for algebra.”**

Bill McCullam, CCSS Mathematics Author,



  Accountability and Curriculum Policy Office  
Preparation for Assessment for Change  
8/11/2011 • page 112

Thus the statement from Dr. McCallum, CCSS author.

Slide 113

**Time to Reflect**

Summary	

  Accountability and Curriculum Policy Office  
Preparation for Assessment for Change  
8/11/2011 • page 113

reflect

Slide 114

© 2001 Randy Glasbergen. www.glasbergen.com

$\$80,000 + 0 = \$800,000$   
Zero pay raise for the CEO who is bad at Math.



**“That’s right, I’ve decided to give myself zero pay raise this year.”**



  Accountability and Curriculum Policy Office  
Preparation for Assessment for Change  
8/11/2011 • page 114

Share cartoon

Slide 115

**Standards for Mathematical Practices**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

  Assessment and Curriculum Review Office  
University of Wisconsin-Madison 8/11/2011 • page 115

Survey group for familiarity.

Refer task completed in algebra

- Find this in your standards
- Carry across all grade levels
- Describe habits of a mathematically expert student
- The Common Core proposes a set of Mathematical Practices that all teachers should develop in their students. These practices are similar to the mathematical processes that NCTM addresses in the Process Standards in *Principles and Standards for School Mathematics*.
- These can be implemented **now** even before content goes into effect.

Slide 116

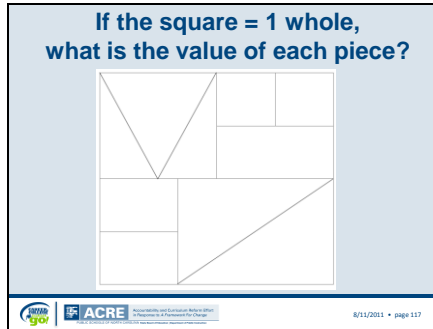
**Fractions**

  Assessment and Curriculum Review Office  
University of Wisconsin-Madison 8/11/2011 • page 116

We will begin to touch on 3-5 fractions today – **just touch this concept** there is no way we can get through all the fraction standards, this is a huge part of 3-5 standards.

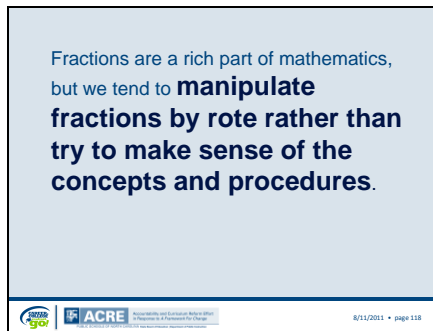
The word “fractions” does not appear in the standards. The new word is “partitions in equal parts”

Slide 117



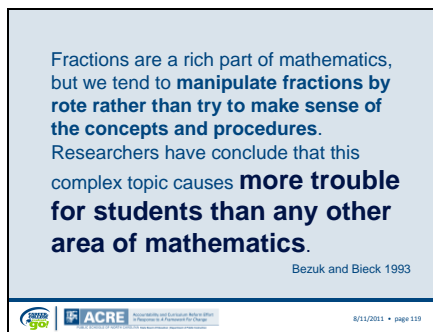
Task for after lunch, break, or morning this handout is in their folder.  
How could this task be differentiated?  
What mathematical practices did you use when doing this task?

Slide 118



Emphasize the bold in the sentence!

Slide 119



Emphasize the bold in the sentence!

Slide 120

**Turn and Talk**

**Where are fractions in the K-2 standards and how are they represented?**

8/11/2011 • page 120

Participants will work at their tables to explore k-2 fractions

Slide 121



timer

8/11/2011 • page 121

Timer if needed

Slide 122

**Geometry** **1.G**

**Reason with shapes and their attributes.**

**1.G.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates **smaller shares**.**

8/11/2011 • page 122

Reiterate what participants hopefully found out about fractions in K-2  
**Partition** - students are expected to use this word in their vocabulary.


Slide 123

**Geometry** **1.G**

**Reason with shapes and their attributes.**

**1.G.3** Partition circles and rectangles into two and four **equal** shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

How can you and a friend share equally (partition) a piece of paper so that you both have the same amount of paper?

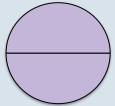


ACRE Accessibility and Customization Policy Sheet  
In Response to a Request for Change  
8/11/2011 • page 123

Share example

Slide 124

**There is a pizza for dinner. What do you notice about the slices on the pizza?**



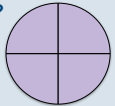
**“There are two slices on the pizza. Each slice is the same size. Those are big slices!”**

ACRE Accessibility and Customization Policy Sheet  
In Response to a Request for Change  
8/11/2011 • page 124

Share example of first grade standard

Slide 125

**If we cut the same pizza into four slices (fourths), do you think the slices would be the same size, larger, or smaller?**



**“When you cut the pizza into fourths. The slices are smaller. More slices mean that the slices get smaller and smaller.”**

ACRE Accessibility and Customization Policy Sheet  
In Response to a Request for Change  
8/11/2011 • page 125



Share example of first grade standard

Slide 126

**Geometry** **2.G**

Reason with shapes and their attributes.

**2.G.3** Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words **halves**, **thirds**, **half of**, **a third of**, etc., and describe the whole as **two halves**, **three thirds**, **four fourths**. Recognize that **equal shares** of identical wholes need not have the same shape.

  Accountability and Curriculum Policy Office  
Preparation for Assessment for Change  
8/11/2011 • page 126

Review second grade standard  
Thirds begin in grade 2 (two, three or four equal squares)

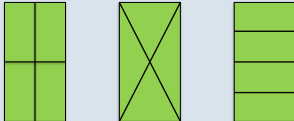
Slide 127



**Geometry** **2.G**

Reason with shapes and their attributes.

**2.G.3** Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words **halves**, **thirds**, **half of**, **a third of**, etc., and describe the whole as **two halves**, **three thirds**, **four fourths**. Recognize that **equal shares** of identical wholes need not have the same shape.

Divide (partition) each rectangle into fourths a different way.





  Accountability and Curriculum Policy Office  
Preparation for Assessment for Change  
8/11/2011 • page 127

Share example of second grade standard

Slide 128

**Now Let's Do Some Math!**

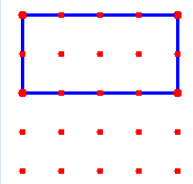
  Accountability and Curriculum Policy Office  
Preparation for Assessment for Change  
8/11/2011 • page 128

Now, Let's do some math.

Slide 129

**Geoboard Fractions**

Make this rectangle on your geoboard.



Find ways to divide the rectangle in halves.

ACRE Accountability and Curriculum Review Office  
Improvement in Assessment for Change  
8/11/2011 • page 129

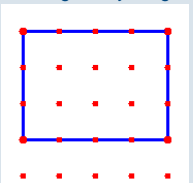
Find at least six ways to divide the rectangle in half, record on your geoboard paper.

- Are the halves the same shape?
- How do you know? (area)
- Are any flipped from the others?

Slide 130

**Geoboard Fractions**

Make this rectangle on your geoboard.



Find ways to divide the rectangle in thirds.

ACRE Accountability and Curriculum Review Office  
Improvement in Assessment for Change  
8/11/2011 • page 130

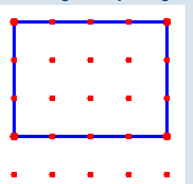
Divide the rectangle in thirds, record on your geoboard paper.

- Are the thirds the same shape?
- How do you know? (area)
- Are any flipped from the others?

Slide 131

**Geoboard Fractions**

Make this rectangle on your geoboard.



Find ways to divide the rectangle in fourths.

ACRE Accountability and Curriculum Review Office  
Improvement in Assessment for Change  
8/11/2011 • page 131

### If time permits

Find ways to divide the rectangle into fourths, record on geoboard paper

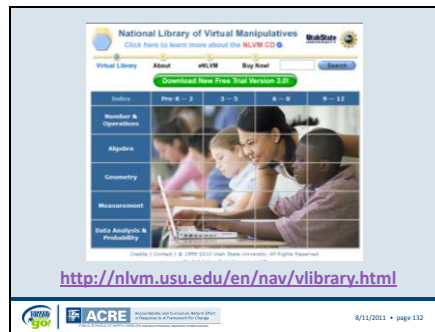
Now -Fourths,

Do you agree with what your partner did, why or why not?

On your geoboard paper, shade a portion on one of your fractions and record symbolically the portion shaded.



Slide 132

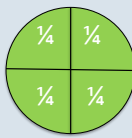


Hyperlink included - Tech standard  
Virtual manipulative site

Slide 133

**Number and Operations—Fractions<sup>5</sup>** **3.NF**

**Reason with shapes and their attributes.**  
**3.G.2 Partition** shapes into parts with **equal areas**. Express the area of each part as a **unit fraction** of the whole.  
*For example, partition a shape into 4 parts with equal area, and describe the area of each part as  $\frac{1}{4}$  of the area of the shape.*

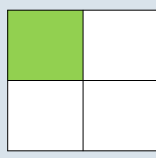


The footer contains logos for 'middle school math' and 'ACRE', the date '8/11/2011', and 'page 133'.

Third grade first time students write fractions symbolically, big focus on unit fractions – foundational for fractions!

Slide 134

Write a fraction to show how much of the large square is shaded.

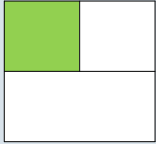




The footer contains logos for 'middle school math' and 'ACRE', the date '8/11/2011', and 'page 134'.

Most students know and can explain  $\frac{1}{4}$   
Resource book *Beyond Pieces and Pies*

Slide 135

Write a fraction to show how much of the large square is shaded.



  Assessment and Curriculum Policy Office  
University of Minnesota - Rochester 8/11/2011 • page 135

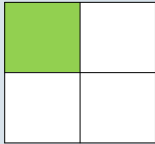
Shocked by responses, most students (9% of upper elementary students) responded  $\frac{1}{3}$ . “First I counted the number of shared parts (one) and used that for the numerator” “Just like I did the other one, only this time it’s one out of three instead of one out of four.” Sometimes our instruction makes students lose that natural instinct students have to partition and understand the fair share piece of fractions because of the procedural way we teach fractions – with no meaning!

**Must provide opportunities for students to work with unequally partitioned areas and number lines.**



That’s why the partition piece so important in the 1-2 grades. This corresponds to the partitive meaning of 1 divided by 4 (next slide)

Slide 136

Write a fraction to show how much of the large square is shaded.



$1 \div 4$

  Assessment and Curriculum Policy Office  
University of Minnesota - Rochester 8/11/2011 • page 136

Most students can explain  $\frac{1}{4}$  which leads into division which is the inverse operation of multiplication

## Slide 137

First Grade	Second Grade	Third Grade
<b>1.G.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words <i>halves</i>, <i>fourths</i>, and <i>quarters</i>, and use the phrases <i>half of</i>, <i>fourth of</i>, and <i>quarter of</i>. Describe the whole as <i>two of</i>, or <i>four of</i> the shares. Understand for these examples that <b>decomposing</b> into more equal shares creates smaller shares.</b>	<b>2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words <i>halves</i>, <i>thirds</i>, <i>half of</i>, <i>a third of</i>, etc., and describe the whole as <i>two halves</i>, <i>three thirds</i>, <i>four fourths</i>. Recognize that equal shares of identical wholes need not have the same shape.</b>	<b>3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a <b>unit fraction</b> of the whole.</b>

8/11/2011 • page 137

Refer to standards and how they build from 1<sup>st</sup> to 3<sup>rd</sup>

## Slide 138

### Part-whole Relationship Research

- Partitioning wholes into equal-size pieces
- Identifying different units

**This is the best way to approach learning about fractions in the early grades. It is essential for students to be provided opportunity to reason about the meaning of part-whole relations.**

8/11/2011 • page 138

**Research supports the idea that part-whole relationship, which involve partitioning wholes into equal-size pieces and identifying different units, is the best way to approach learning about fractions in the early grades.**

students' experiences with "fair shares" in everyday life is often the starting point for assisting students in understanding some important ideas about size and number of units in a whole and how units can be divided up into smaller and smaller subunits

## Slide 139

### Measurement and Data 4.MD

**Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**

**4.MD.1** Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a **larger unit in terms of a smaller unit**. Record measurement equivalents in a two column table.

*For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*



8/11/2011 • page 139

Example of how larger items are partitioned into smaller pieces in the measurement domain.

Slide 140

**Symbolic representation of fractions**

Instead of viewing a number such as 34 as representing a **specific quantity**, when the same digits (3 and 4) are used in the number  $\frac{3}{4}$ , the digits represent a **relationship**.

  Assessment and Curriculum Review Office  
University of Wisconsin-Madison 8/11/2011 • page 140

- Students must be able to think about numbers in a different way than when they are working with whole numbers.
- Instead of viewing a number such as 34 as representing a specific quantity, when the same digits (3 and 4) are used in the number  $\frac{3}{4}$ , the digits represent a relationship.
- Students often find fractions challenging because the principles and procedures for working with fractions differ in important ways from the principles and procedures for working with whole numbers.

Slide 141

**Number and Operations—Fractions<sup>5</sup>** **3.NF**



**Develop understanding of fractions as numbers.**

**3.NF.1** Understand a fraction  $\frac{1}{b}$  as the quantity formed by 1 part when a whole is partitioned into  $b$  equal parts; understand a fraction  $\frac{a}{b}$  as the quantity formed by  $a$  parts of size  $\frac{1}{b}$ .

**3.NF.2** Understand a fraction as a number on the **number line**; represent fractions on a number line diagram.

a. Represent a fraction  $\frac{1}{b}$  on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into  $b$  equal parts. Recognize that each part has size  $\frac{1}{b}$  and that the endpoint of the part based at 0 locates the number  $\frac{1}{b}$  on the number line.

b. Represent a fraction  $\frac{a}{b}$  on a number line diagram by marking off  $a$  lengths  $\frac{1}{b}$  from 0. Recognize that the resulting interval has size  $\frac{a}{b}$  and that its endpoint locates the number  $\frac{a}{b}$  on the number line.

©2010 by Education. All rights reserved. All trademarks are the property of their respective owners.   Assessment and Curriculum Review Office  
University of Wisconsin-Madison 8/11/2011 • page 141

This is not intended to be read - Use this slide as a reference – participants should refer to their CCSS.

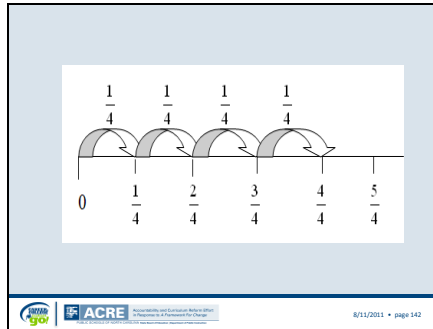
Number line is partitioned. Use area models only in third grade!

As you and teachers around the state read these standards, understanding them may be a challenge. It will take time and conversation.

Turn and Talk – what do these standards mean to you? You may want to draw a model to share your understanding or to communicate your thinking or justify your reasoning with your partner.

Teachers are going to need opportunities to do this same kind of thing. Discuss the meaning of standards – Think about when and where will this opportunity be provided?

Slide 142



Reiterate the meaning of the standard - clarify.

Standard is not that much different than what we are currently doing (but clearer expectations) but it is written in an entirely different manner. Written by mathematicians not elementary educators.

Slide 143

**What happens to the value of the fraction if....**

- Numerator is increased by 1?
- Denominator is decreased by 1?
- Denominator is increased by 1?

Possible questions for students

Some questions teacher could possibly ask that would assist students in clarifying their thinking, reasoning, and understanding of fractions.

Turn and talk if time permits or just share

Slide 144

**Understanding**

- Avoids too much emphasis on procedure
- Facilitates flexibility
- Justified conclusions
- Application to practical situations
- Accurate and clear expectations
- Metacognition

Understanding in the CCSS could mean.....

Common Core is **NOT** written with any one taxonomy. Not revised Blooms, if it is used then it could lead to misinterpretation of the standards. Understand in RBT means construct meaning.

DPI mathematics consultants gets phone calls about this often so please help clarify this misunderstanding out in the field!

## Slide 145



**Number and Operations—Fractions<sup>5</sup>**
**3.NF**

**Develop understanding of fractions as numbers.**

**3.NF.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

- Understand two fractions as equivalent (equal) if they are the same size, or the same point on a **number line**.
- Recognize and generate simple equivalent fractions, e.g.,  $\frac{1}{2} = \frac{2}{4}$ ,  $\frac{4}{6} = \frac{2}{3}$ . Explain why the fractions are equivalent, e.g., by using a visual fraction model.
- Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form  $3 = \frac{3}{1}$ ; recognize that  $\frac{6}{1} = 6$ ; locate  $\frac{4}{4}$  and 1 at the same point of a number line diagram.*
- Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

<sup>5</sup> Grade 3 expectations in this domain are limited to fraction with denominators 2, 3, 4, 6, and 8

Accountability and Curriculum Policy Office  
Arkansas Department of Education  
Arkansas State Capitol, Little Rock, AR 72201  
www.ade.state.ar.us

8/11/2011 • page 145



- This slide is not meant to be read but used as a reference
- Turn and Talk –use manipulatives on table
- Next slide questions Using chart paper – how would you teach the assigned standard?
- Which standard is new or different than what third graders currently learn?
- Again – this conversation it vital in grade level meeting or PLC's
- Do a walk-about to see how standards could be taught.
- Points to highlight:
- Specially mentions number line. Will talk more about this later!
- Footnote limited to 2, 3, 4, 6, and 8
- Fraction models in third grade include area (parts of a whole) models (circles, rectangles, squares) and number lines. Set models (part of a group) are not introduced in third grade. Students should focus on the concept that a fraction is made up (compose) of many pieces of a unit fractions, which has a numerator of 1. for example, the fraction  $\frac{3}{5}$  is composed of 3 pieces that each has a size of  $\frac{1}{5}$ .
- This is not related (family) fractions must compare all fractions

## Slide 146

**Turn and Talk**

**Third Grade Fractions**

- What math tools could you use to teach third grade fractions?
- What are teachers currently using?
- What will be a challenge for them when teaching fractions in third grade?
- How will we help them overcome these challenges?

Accountability and Curriculum Policy Office  
Arkansas Department of Education  
Arkansas State Capitol, Little Rock, AR 72201  
www.ade.state.ar.us

8/11/2011 • page 146

Using chart paper – how would you teach 3.NF.3 standard?

Which standard is new or different than what third graders currently learn?

Again – this conversation it vital in grade level meetings or PLC's

Post Do a walk-about at break to see how standards could be taught.

Slide 147



Timer if needed

Slide 148

**Number Line**

**Featured prominently in the Common Core Content Standards as a model for representing numbers**

Not only does use of the number lines persist across grade levels, but also across domains.

8/11/2011 • page 148

- It first appears in the Measurement and Data domain in second grade wherein students are expected to represent numbers as lengths on the number line as well as sums and differences on a number line.
- Number line references are numerous in grade three, particularly in the Number and Operations domain, but in Measurement and Data as well.
- Reliance on the number line continues in grade four; references are seen through grade eight as well as in the high school Statistics and Probability domain.
- Not only does use of the number persist across grade levels, but also across domains.
- The number line serves as a visual /physical model to represent the counting numbers and constitutes an effective tool to develop estimation techniques, as well as a helping instrument when solving word problems.
- The number line constitutes a unifying and coherent representation for the different sets of numbers (N, Z, Q, R) which the other models cannot do.
- The number line is an appropriate model to make sense of each set of numbers as an expansion of other and to build the operations in a coherent mathematical way.
- The number line enables to present the fractions as numbers and to explore the notion of equivalent fractions in a meaningful way.
- The number line, in some way, looks like a ruler, fostering the use of the metric system



and the decimal numbers.

- The number line fosters the discovery of the density property of rational numbers.
- The number line provides an opportunity to consider numbers that are not fractions and consider the existence of irrational numbers.
- Share rulers – Master Rulers

Slide 149

### Number Line

- At your table is an envelope with information about the number line.
- Each person at the table is to take a slip of paper from the envelope, read it, and reflect.
- Then share information to others at your table.



ACRE

Assessment and Continuous Review Office  
University of Wisconsin-Madison

8/11/2011 • page 149

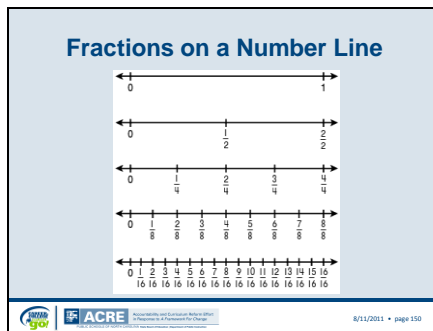
- It first appears in the Measurement and Data domain in At your table is an envelope with information about the number line.
- Each person at the table is to take a slip of paper from the envelope, read it, and reflect.
- Then share information to others at your table.
- Second grade where in students are expected to represent numbers as lengths on the number line as well as sums and differences on a number line.
- Number line references are numerous in grade three, particularly in the Number and Operations domain, but in Measurement and Data as well.
- Reliance on the number line continues in grade four; references are seen through grade eight as well as in the high school Statistics and Probability domain.
- Not only does use of the number persist across grade levels, but also across domains.
- The number line serves as a visual /physical model to represent the counting numbers and constitutes an effective tool to develop estimation techniques, as well as a helping instrument when solving word problems.
- The number line constitutes a unifying and coherent representation for the different sets of numbers (N, Z, Q, R) which the other models cannot do.
- The number line is an appropriate model to make sense of each set of numbers as an expansion of other and to build the



operations in a coherent mathematical way.

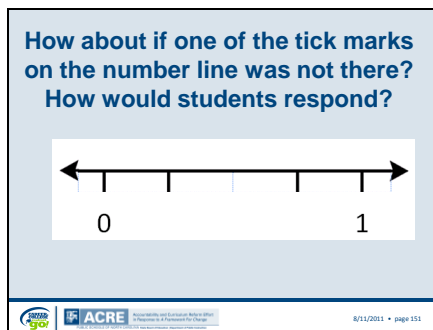
- The number line enables to present the fractions as numbers and to explore the notion of equivalent fractions in a meaningful way.
- The number line, in some way, looks like a ruler, fostering the use of the metric system and the decimal numbers.
- The number line fosters the discovery of the density property of rational numbers.
- The number line provides an opportunity to consider numbers that are not fractions and consider the existence of irrational numbers.

Slide 150



When looking at these number lines, what observations do you make?

Slide 151



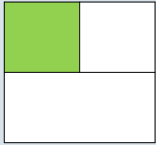
How about if one of the tick marks weren't there?

How would students respond?

Goes back to the partitioning in 1-3. (next slide)

Slide 152

**Write a fraction to show how much of the large square is shaded.**

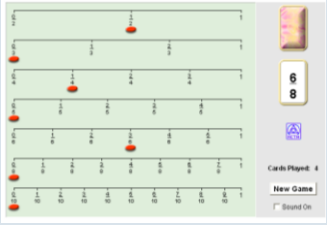


8/11/2011 • page 152

How is this area model like the number line we just saw?

Slide 153

**Fraction Game**



<http://illuminations.nctm.org/ActivityDetail.aspx?ID=18>

8/11/2011 • page 153

Hyperlink to NCTM inactive site

Slide 154

**Now Let's Do Some Math!**



8/11/2011 • page 154

Now, Let's do some math.

Slide 155

### Fraction Sort

- Sort the fraction cards
- Fractions equal to or close to 0
- Fractions equal to or close to 1
- Fractions equal to or close to  $\frac{1}{2}$

Accountability and Curriculum Policy Office  
Preparation for Assessment for Change

8/11/2011 • page 155

- How did you sort them, why did you sort them that way?
- Now put them in order from smallest to largest. Justify your reasoning
- Now, as a whole group get into a line in front of the room with large fraction cards, model whole group discussion.

Slide 156



### Fraction Estimation

- Add these fractions in your sort:

$\frac{5}{9}$  $\frac{1}{12}$  $\frac{17}{20}$

- Estimate the sum:

$\frac{4}{5}$  $\frac{7}{12}$  $\frac{1}{8}$  $\frac{8}{9}$

Accountability and Curriculum Policy Office  
Preparation for Assessment for Change

8/11/2011 • page 156



Where did you put the fractions in your sort and why?  
Talk me through how you estimated the sum of the two fractions?

Slide 157

### Estimate the answer to $(12/13) + (7/8)$

A. 1  
B. 2  
C. 19  
D. 21

Only 24% of 13 year olds answered correctly.  
Equal numbers of students chose the other answers.

Accountability and Curriculum Policy Office  
Preparation for Assessment for Change

8/11/2011 • page 157

Data from NAEP  
Only 24% of 13-year olds answered correctly.  
Equal numbers of students chose the other answers.

Slide 158

**Fractions in Context**

**When pitching, Joe struck out 7 of the 12 batters**

- Exactly half =  $\frac{1}{2}$
- About half =  $\frac{1}{2}$
- Less than half <  $\frac{1}{2}$
- More than half >  $\frac{1}{2}$

**Sally blocked 5 goals out of 8 attempt**  
**Of the 100 coins in Jim's bank, 34 were pennies**  
**Come up with a few fractions in context with your partner.**

8/11/2011 • page 158

- Read the first statement. “Do you think Joe struck out exactly half, about half, less than half, or more than half of the batters?” Explain your reasoning!
- Write your own scenarios.
- Twenty-five students in the class have pets. Twelve of them have dogs. Nine have cats. Six have fish.
- As much as possible, it’s helpful to relate student’s study of mathematics to how we use and talk about mathematics outside of school. Not only does this help students see mathematics as useful and related to real-world situations, but it also helps them bring meaning to abstract concepts.
- This task is useful for introducing students to estimating with fractions and gives them experience with the idea that the precision required for answers to mathematical problems depends on the situation and the purpose for the communication.

Slide 159

**Number and Operations—Fractions** **4.NF**

**Extend understanding of fraction equivalence and ordering.**

**4.NF.1** Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

**4.NF.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $1/2$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.



8/11/2011 • page 159

- “Recognize that comparisons are valid only when the two fractions refer to the same whole”
- Could share task if time permits
- Who wants half of a Mr. Good Bar? (miniature size)
- Who wants half of a Heresy Bar (Giant size bar)
- Is a half always equal to a half?
- Who wants to partition a candy bar into halves?
- Which tasks align best with the understanding and thinking expectations of this standard?
- How could this task be modified?

Slide 160

### Equivalent

- There are many ways to name the same number.
  - 5 can be written as  $1 + 4$  or  $2 + 3$ .
  - $700 + 80 + 3$  can be written as 783.
- There are different names for fractions, too one-third can be written as two-sixths.
- When two fractions name the same number, we say they are equivalent.

  Accountability and Curriculum Policy Office  
Improving Learning Outcomes for All 8/11/2011 • page 160

Folding paper to build equivalents.

Slide 161



### Folding Fractions

Equivalence – many ways to name the same number.

1. Fold a piece of paper into thirds.
2. Unfold, then color one third.
3. Make and record table.

Colored Parts	Total Parts	Fraction Colored
1	3	$\frac{1}{3}$

4. Now fold the paper in half the other way.
5. Record your results on the table.
6. Explore other possible equivalent fractions.



  Accountability and Curriculum Policy Office  
Improving Learning Outcomes for All 8/11/2011 • page 161

- Folding paper  $\frac{1}{3}$  in half what fraction do you have?  $\frac{2}{6}$
- Now try it again, fold it in thirds  $\frac{3}{9}$ . Record on table, what did you notice?
- Start a new table and use a new piece of paper  $\frac{1}{2}$  then again with  $\frac{1}{4}$
- Etc...

Slide 162

### What other mathematic tools could teachers use to help students build concrete understanding of equivalent fractions?

- Pattern block
- Fraction bars
- Rods
- Number lines

  Accountability and Curriculum Policy Office  
Improving Learning Outcomes for All 8/11/2011 • page 162

Take 2 minutes - Turn and talk

Slide 163

**Number and Operations—Fractions** 4.NF

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

**4.NF.3** Understand a fraction  $a/b$  with  $a > 1$  as a sum of fractions  $1/b$ .

a. Understand addition and subtraction of fractions as **joining and separating parts** referring to the same whole.

b. **Decompose a fraction** into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples:  $3/8 = 1/8 + 1/8 + 1/8$ ;  $3/8 = 1/8 + 2/8$ ;  $2 \frac{1}{8} = 1 + 1 \frac{1}{8} = 8/8 + 8/8 + 1/8$ .

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

8/11/2011 • page 163

b. – decomposing, brings back emphasizes the unit fraction understanding.

4.NF.3.b Students should justify their breaking apart (decomposing) of fractions using visual fraction models. The concept of turning mixed numbers into improper fractions needs to be emphasized using visual fraction models.

Slide 164

**Decomposing Fractions**

$3/8 = 1/8 + 1/8 + 1/8$

$3/8 = 1/8 + 2/8$

$2 \frac{1}{8} = 1 + 1 \frac{1}{8}$

or

$2 \frac{1}{8} = 8/8 + 8/8 + 1/8$

8/11/2011 • page 164

- Students should justify their breaking apart (decomposing) of fractions using visual fraction models.
- The concept of turning mixed numbers into improper fractions needs to be emphasized using visual fraction models.

Slide 165

**Adding Fractions**

A cake recipe calls for you to use  $3/4$  cup of milk,  $1/4$  cup of oil, and  $2/4$  cup of water. How much liquid was needed to make the cake?

$3/4 + 1/4 + 2/4 = 6/4 = 1 \frac{2}{4}$



8/11/2011 • page 165

Model, pulled from the unpacking document

Slide 166

### Fractions on a Number Line

- What does it look like?
- Work with a partner and model the use of a number line when adding and subtracting fractions.
- Share your model with others at your table.

  Accountability and Curriculum Review Office  
In Response to a Request for Change 8/11/2011 • page 166



Turn and Talk

Slide 167

### Number and Operations—Fractions 4.NF

**4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.**

- Understand a fraction  $a/b$  as a multiple of  $1/b$ . For example, use a visual fraction model to represent  $5/4$  as the product  $5 \times (1/4)$ , recording the conclusion by the equation  $5/4 = 5 \times (1/4)$ .
- Understand a multiple of  $a/b$  as a multiple of  $1/b$ , and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express  $3 \times (2/5)$  as  $6 \times (1/5)$ , recognizing this product as  $6/5$ . (In general,  $n \times (a/b) = (n \times a)/b$ .)
- Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat  $3/8$  of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

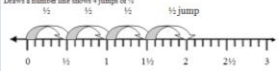
  Accountability and Curriculum Review Office  
In Response to a Request for Change

- Multiplication of fraction by a whole number. This could be a gap from fourth to fifth in first year of implementation.
- In a relay race, each runner runs  $\frac{1}{2}$  of a lap. If there are 4 team members how long is the race?


Slide 168

In a relay race, each runner runs  $\frac{1}{2}$  of a lap. If there are 4 team members how long is the race?

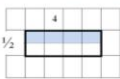
**Student 1**  
Draws a number line showing 4 jumps of  $\frac{1}{2}$ .





**Student 2**  
Draws an area model showing 4 pieces of  $\frac{1}{2}$  joined together to equal 2.



**Student 3**  
Draws an area model representing  $4 \times \frac{1}{2}$  on a grid, dividing one row into  $\frac{1}{2}$  to represent the multiplier.



  Accountability and Curriculum Review Office  
In Response to a Request for Change 8/11/2011 • page 168

- We want student to be exposed to the content/concepts using various models (visual fraction models)
- 4.NF.4.c calls for students to use visual fraction models to solve word problems related to multiplying a whole number by a fraction.
- In a relay race, each runner runs  $\frac{1}{2}$  of a lap. If there are 4 team members how long is the race?
- Number line, tape diagram or linear model, and an area model



## Slide 169

**Number and Operations—Fractions** **5.NF**

**Use equivalent fractions as a strategy to add and subtract fractions.**

**5.NF.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,  $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$ . (In general,  $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$ .)*

**5.NF.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result  $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$ , by observing that  $\frac{3}{7} < \frac{1}{2}$ .*

  Accountability and Curriculum Policy Office  
Florida Department of Education  
8/11/2011 • page 169

- 5<sup>th</sup> grade add and subtract unlike denominators
- How is this different? Explicitly states, “by replacing given fractions with equivalent fractions”



## Slide 170

**Number and Operations—Fractions** **5.NF**

**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**

**5.NF.3** Interpret a fraction as division of the numerator by the denominator ( $\frac{a}{b} = a \div b$ ). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

*For example, interpret  $\frac{3}{4}$  as the result of dividing 3 by 4, noting that  $\frac{3}{4}$  multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size  $\frac{3}{4}$ . If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

  Accountability and Curriculum Policy Office  
Florida Department of Education  
8/11/2011 • page 170

Multiplication make sure you look at the examples provide to help with clarification of standard.

## Slide 171



**Number and Operations—Fractions** **5.NF**

**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**

**5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product  $(\frac{a}{b}) \times q$  as a parts of a partition of  $q$  into  $b$  equal parts; equivalently, as the result of a sequence of operations  $a \times q \div b$ . *For example, use a visual fraction model to show  $(\frac{2}{3}) \times 4 = \frac{8}{3}$ , and create a story context for this equation. Do the same with  $(\frac{2}{3}) \times (\frac{4}{5}) = \frac{8}{15}$ . (In general,  $(\frac{a}{b}) \times (\frac{c}{d}) = \frac{ac}{bd}$ .)*

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

  Accountability and Curriculum Policy Office  
Florida Department of Education  
8/11/2011 • page 171

Multiplication of fractions moved from 6<sup>th</sup> grade.  
Area - building concrete understanding




Slide 172

**Using the Paper Folding  
to Multiply Fractions**

$$\frac{1}{2} \times \frac{1}{3}$$

A third of the class is wearing green. Half of them are boys. How many boys are wearing green?

 8/11/2011 • page 172


- Multiplying fractions means cutting a portion into smaller pieces
- A third of the class is wearing green. Half of them are boys. How many boys are wearing green?


Slide 173

**Multiplication of Fractions**

Two-fifths of the employees at a very large company has Type A blood. If  $\frac{1}{5}$  of the company's employees donate blood what fraction will donate type A blood.

Blue = company



 8/11/2011 • page 173

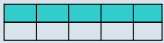
The question asks - what is  $\frac{2}{5}$  of  $\frac{1}{5}$ , area model  $\frac{1}{5}$  - building concrete understanding not moving straight to the algorithm.

Slide 174

### Multiplication of Fractions

Two-fifths of the employees at a very large company has Type A blood. If  $\frac{1}{2}$  of the company's employees donate blood what fraction will donate type A blood.

Blue = company



8/11/2011 • page 174

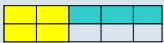
Model now  $\frac{1}{2}$  is divided into fifths

Slide 175

### Multiplication of Fractions

Two-fifths of the employees at a very large company has Type A blood. If  $\frac{1}{2}$  of the company's employees donate blood what fraction will donate type A blood.

Blue = company  
Yellow = Employees with Type A blood



8/11/2011 • page 175

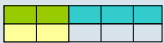
Model showing  $\frac{2}{5}$  shaded

Slide 176

### Multiplication of Fractions

Two-fifths of the employees at a very large company has Type A blood. If  $\frac{1}{2}$  of the company's employees donate blood what fraction will donate type A blood.

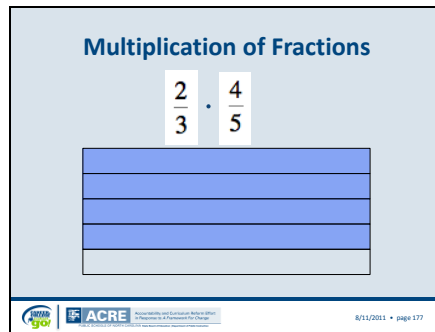
Blue = company  
Yellow = Employees with Type A blood



8/11/2011 • page 176

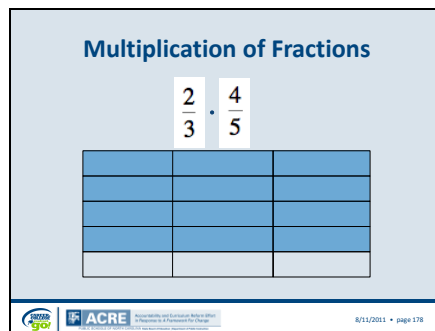
Model showing overlapping part as solution to problem.

Slide 177

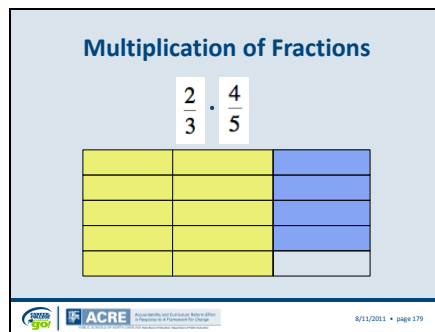


Another example talking through what's happening.  
 $\frac{2}{3}$  of  $\frac{4}{5}$   
 Model divided into 5 equal part 4 shaded

Slide 178

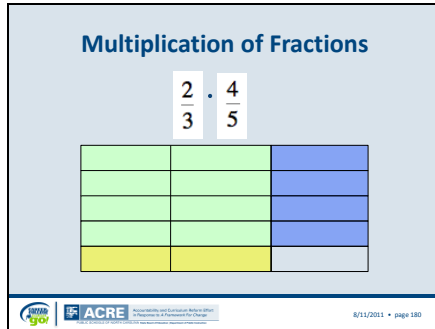


Slide 179



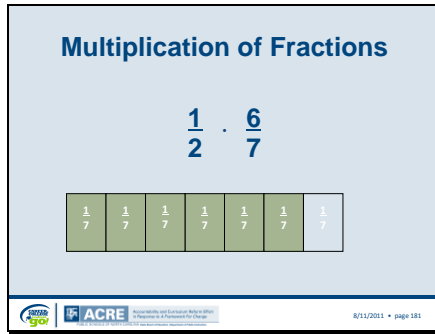
Fifths divided into 3 equal parts, 2 of the three parts shaded

Slide 180



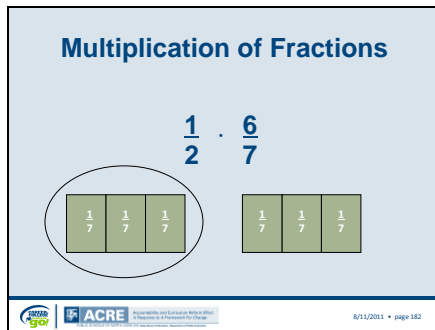
Overlapping portion is the solution to the problem

Slide 181



Another example - model

Slide 182



Model shows  $\frac{1}{2}$  of  $\frac{6}{7}$

Slide 183

Three-fourths of the class is boys. Two-thirds of the boys are wearing tennis shoes. What fraction of the class are boys with tennis shoes?

This question is asking what is  $\frac{2}{3}$  of  $\frac{3}{4}$  or what is  $\frac{2}{3} \times \frac{3}{4}$ .

Student 2

8/11/2011 • page 183

Number line another model for students, must be expose to various models

Slide 184

**Number and Operations—Fractions** **5.NF**

**5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.<sup>1</sup>**

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for  $(\frac{1}{3}) \div 4$ , and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that  $(\frac{1}{3}) \div 4 = \frac{1}{12}$  because  $(\frac{1}{12}) \times 4 = \frac{1}{3}$ .

b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for  $4 \div (\frac{1}{5})$ , and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that  $4 \div (\frac{1}{5}) = 20$  because  $20 \times (\frac{1}{5}) = 4$ .

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share  $\frac{1}{2}$  lb of chocolate equally? How many  $\frac{1}{3}$ -cup servings are in 2 cups of raisins?

8/11/2011 • page 184

Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

Slide 185

**Division of Fractions**

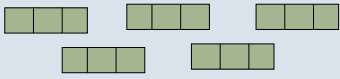
$5 \div \frac{1}{3} = ?$

8/11/2011 • page 185

Dividing fractions mean determining how many smaller pieces there are in a larger piece.  
Context?  
5 candy bars are divided into thirds, how many people will get a piece of the candy bar?

Slide 186

**Division of Fractions**

$$5 \div \frac{1}{3} =$$


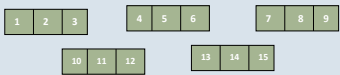
ACRE Accessibility and Evaluation Policy Sheet  
Inquiries to: ACRE@fda.gov

8/11/2011 • page 186

5 candy bars divided into thirds

Slide 187

**Division of Fractions**

$$5 \div \frac{1}{3} =$$


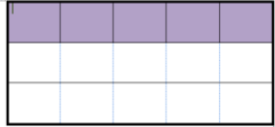
ACRE Accessibility and Evaluation Policy Sheet  
Inquiries to: ACRE@fda.gov

8/11/2011 • page 187

How many people received a third of the candy bar?

Slide 188

**Division of Fractions**

$$\frac{1}{3} \div 5 =$$


ACRE Accessibility and Evaluation Policy Sheet  
Inquiries to: ACRE@fda.gov



8/11/2011 • page 188

$\frac{1}{3}$  of the sheet cake is shared among 5 people so each person get  $\frac{1}{15}$ <sup>th</sup> of the sheet cake

Slide 189

**How is  $1/3 \div 5$  different?**



- *Use the relationship between multiplication and division to explain that  $(1/3) \div 5 = 1/15$  because  $(1/15) \times 5 = 1/3$ .*
- *Create a story context*

  Accountability and Curriculum Policy Office  
Preparation for a Standard for Change 8/11/2011 • page 189

Multiplying fractions means cutting a portion into smaller portions.  
Dividing fractions means determining how many smaller pieces there are in a larger piece.

Slide 190



**How would this look on a number line?**

  Accountability and Curriculum Policy Office  
Preparation for a Standard for Change 8/11/2011 • page 190

Explore numberline

Slide 191

**“The value of the common core is only as good as the implementation of the mathematical practices.”**  
-- Jere Confrey



  Accountability and Curriculum Policy Office  
Preparation for a Standard for Change 8/11/2011 • page 191

- We would like to leave you with this profound statement by Jere Confrey
- The value of the common core is only as good as the implementation of the mathematical practices.

Slide 192

### Time to Reflect

Summary	

  Accountability and Continuous Improvement Office  
In Response to a Movement for Change 8/11/2011 • page 192

reflect

Slide 193



POST-GRAD: "WHAT'S 12 x 7?"  
A BILLION THIRTEENS!  
MAT: A MISTAKE THAT CAN'T BE RIGHT...  
MAT'S MISTAKE SHE SAID 3 x 4 WAS...

  Accountability and Continuous Improvement Office  
In Response to a Movement for Change 8/11/2011 • page 193

share

Slide 194



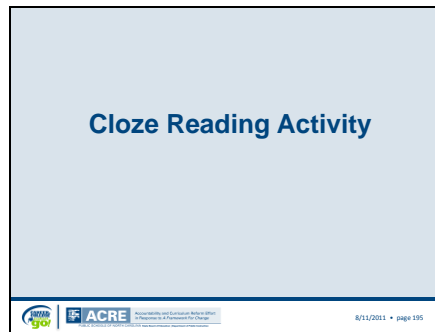
### Questions and Comments

  Accountability and Continuous Improvement Office  
In Response to a Movement for Change 8/11/2011 • page 194

questions

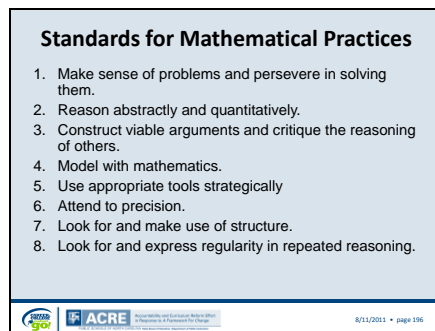


Slide 195



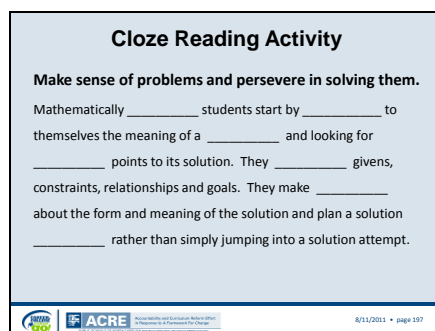
Close reading

Slide 196



- Carry across all grade levels
- Describe habits of a mathematically expert student
- The Common Core proposes a set of Mathematical Practices that all teachers should develop in their students. These practices are similar to the mathematical processes that NCTM addresses in the Process Standards in *Principles and Standards for School Mathematics*.
- These can implemented between now and time content goes into effect.

Slide 197





Close reading

Slide 198

### Cloze Reading Activity

**Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt.

Accountability and Curriculum Policy Office  
In Response to a Request for Change  
©2011 National Council of State Mathematics8/11/2011 • page 198



solutions

Slide 199

### Cloze Reading Activity

**Construct viable arguments and critique the reasoning of others.**

Mathematically \_\_\_\_\_ students understand and \_\_\_\_\_ stated assumptions, definitions, and previously established results in constructing \_\_\_\_\_. They make conjectures and build a logical progression of \_\_\_\_\_ to explore the \_\_\_\_\_ of their conjectures.

Accountability and Curriculum Policy Office  
In Response to a Request for Change  
©2011 National Council of State Mathematics8/11/2011 • page 199



Close reading

Slide 200

### Cloze Reading Activity

**Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures.

Accountability and Curriculum Policy Office  
In Response to a Request for Change  
©2011 National Council of State Mathematics8/11/2011 • page 200

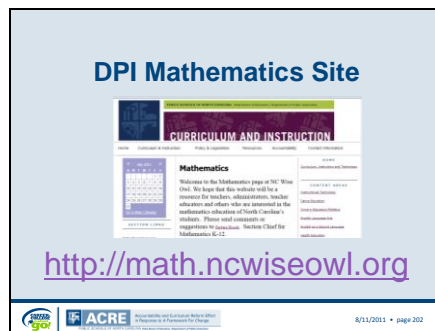
solutions

Slide 201



- We cannot teach mathematics the way we were taught.
- Technology won't allow it. Today's kids are different because of it!
- We know much more about how children learn mathematics.
- “If we keep doing what we always have done we will keep getting what we have always gotten!”

Slide 202



Hyperlink to share resources

Slide 203





Adopted in June, 2010  
Goes into effect 2012-2013

Slide 204

### Something to Think About

- What if we didn't have a requirement for math – how would we lure students in?

-- Jere Confrey



  Assessment and Evaluation Policy Office  
Preparation for Assessment for Change 8/11/2011 • page 204

Think about

Slide 205

### Time to Reflect

Summary	


  Assessment and Evaluation Policy Office  
Preparation for Assessment for Change 8/11/2011 • page 205

reflect

Slide 206



"I think that's right, but let me check."

  Assessment and Evaluation Policy Office  
Preparation for Assessment for Change 8/11/2011 • page 206

Share cartoon

Slide 207



## Questions



  Accountability and Continuous Improvement Office  
Preparation for Assessment for Change  
8/11/2011 • page 207

questions

Slide 208

## Contact Information

Kitty Rutherford Mathematics Consultant 919-807-3934 <a href="mailto:kitty.rutherford@dpi.nc.gov">kitty.rutherford@dpi.nc.gov</a>	Amy Scrinzi Mathematics Consultant 919-807-3934 <a href="mailto:amy.scrinzi@dpi.nc.gov">amy.scrinzi@dpi.nc.gov</a>
Barbara Bissell K-12 Mathematics Section Chief 919-807-3838 <a href="mailto:barbara.bissell@dpi.nc.gov">barbara.bissell@dpi.nc.gov</a>	
Joyce Gardner ERD Consultant 828-242-9872 <a href="mailto:joyce.gardner@dpi.nc.gov">joyce.gardner@dpi.nc.gov</a>	Gerri Batchelor IT Consultant 919-807-3449 <a href="mailto:gerri.batchelor@dpi.nc.gov">gerri.batchelor@dpi.nc.gov</a>



  Accountability and Continuous Improvement Office  
Preparation for Assessment for Change  
8/11/2011 • page 208

Contact information

Slide 209

## Resources Referenced

- Beyond Pizzas and Pies  
by McNamara and Shaughnessy
- Children's Mathematics, Cognitively Guided Instruction  
by Carpenter, Fennema, Franke, Levi & Empson
- Developing Number Concepts (Books 1-3)  
by Kathy Richardson
- Formative Assessment  
by Margaret Heritage
- Teaching Student-Centered Mathematics, K-3; 3-5  
by Van de Walle and Lovin
- Young Mathematicians at Work  
by Fosnot and Dolk

  Accountability and Continuous Improvement Office  
Preparation for Assessment for Change  
8/11/2011 • page 209

Resources referred to in PD